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An integrated value-derivative model for the steel industry to evaluate and optimize the impact of operational strategies using total enterprise performance indicators

Jaime Alberto Torres
University of Tennessee

To the Graduate Council:

I am submitting herewith a dissertation written by Jaime Alberto Torres entitled "An integrated value-derivative model for the steel industry to evaluate and optimize the impact of operational strategies using total enterprise performance indicators." I have examined the final electronic copy of this dissertation for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, with a major in Engineering Science.

Charles H. Aikens, Major Professor

We have read this dissertation and recommend its acceptance:

Adedeji B. Badiru, Kenneth E. Kirby, Dukwon Kim, Chris D. Cox

Accepted for the Council:

Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

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AN INTEGRATED VALUE-DERIVATIVE MODEL FOR THE STEEL INDUSTRY TO EVALUATE AND OPTIMIZE THE IMPACT OF OPERATIONAL STRATEGIES USING TOTAL ENTERPRISE PERFORMANCE INDICATORS

A Dissertation
Presented for the
Doctor of Philosophy Degree
The University of Tennessee, Knoxville

Jaime Alberto Torres
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DEDICATION

This dissertation is dedicated to my parents, Doris y Jaime; to my brother, Carlos Mario; to my wife, Judith Omaira; and to my son, David Enrique, for always believing in me, inspiring me, and encouraging me to reach higher in order to achieve my goals.

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Finally, my parents, Jaime and Doris; my brother, Carlos Mario; my wife, Judith Omaira; and my son, David Enrique, deserve my special thanks for their continued and unconditional encouragement, patience, sacrifice, and love. Without them, I would not have been able to complete this research.

ABSTRACT

The purpose of this research was to develop a structured evaluation and optimization methodology for a prototype Value Chain Analysis model created by the Oak Ridge National Laboratory to identify and select new operational strategies/technologies for steel manufacturing plants in order to enhance their performance.

The research's major objectives were (a) to develop an enterprise mathematical model that describes the steel manufacturing process in terms of performance indicators, that adequately explains the marginal changes in outputs that occur per unit changes in inputs at the process step level, and that further illustrates how each process chains together in the production sequence; (b) to develop enterprise mathematical programming models for a number of optimization approaches to search for optimal or pareto-optimal values of the process performance indicators given a set of parameters; and (c) to develop methods to numerically solve, through a mix of heuristic and optimization techniques, the mathematical programming problems to optimize the manufacturing process' performance in order to achieve the maximum leveraged benefits for the entire enterprise.

A detailed presentation of the theoretical model development process is provided, including in some cases numerical examples to illustrate the mathematical formulations and two comprehensive numerical examples to illustrate and validate the proposed solution methodologies for the enterprise mathematical programming models. Furthermore, recommendations for further research are discussed.

PREFACE

"The objective of optimization is to select the best possible decision for a given set of circumstances without having to enumerate all the possibilities. Three basic components are required to optimize an industrial process. First, a mathematical model for the process must be available, and the process variables that can be manipulated and controlled must be known. Secondly, an economic model of the process is required. This is an equation that represents the profit made from the sale of products and the costs associated with their production. Finally, the optimization procedure selected must locate the values of the independent variables of the process to produce the maximum profit or minimum cost as measured by the economic model. Also, the constraints in materials, process equipment, manpower, etc. must be satisfied as specified by the process model."

Ralph Pike. *Optimization for Engineering Systems*. Van Nostrand Reihnold: New York, 1986.

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LIST OF SYMBOLS

SUBSCRIPT SYMBOLS

$a = 1,2,\ldots,q$	indicates operational strategy.
$i = 1,2,\ldots,n$	indicates final product.
$h, j, k, l = 0,1,2,\ldots,m$	indicates process (Process 0 represents raw material supply).

VARIABLE SYMBOLS

AE_a	Expected annual expenses for a^{th} new operational strategy (\$/year).
AW_a	Annual worth measure for a^{th} new operational strategy (\$/year).
$(A/P,\text{i\%},N)$	Capital recovery factor.
B	Enterprise's available investment budget (\$).
C_j	Total accumulated cost per ton of material output after the j^{th} process (\$/ton).
$\hat{C}_j$	Reduced total accumulated cost per ton of final product after the j^{th} process due to the implementation of an operational strategy (\$/ton).
C'_l	Reduced total accumulated cost per ton of ΔP_j that becomes a given $Ti_{(l)}$ when C_j is made equal to zero (\$/ton).
CF_a	Annual cash flow for a^{th} new operational strategy (\$/year).
$\left(\frac{\Delta C}{\Delta P}\right)_j$	Cost-to-product sensitivity parameter for j^{th} process (\$/ton).
E	Enterprise's total energy consumption (kW·h/year).
Em	Enterprise's maximum total energy consumption (kW·h/year).
E_j	Total energy consumed by j^{th} process (kW·h/year).
Et_j	Energy consumed per ton arriving to j^{th} process (kW·h/ton).
EB_j	Economic benefit for j^{th} process (\$/year).
EBC_j	Economic benefit coefficient for j^{th} process (\$/ton).
EBC^*_j	Economic benefit coefficient for j^{th} process after the introduction of an optimum set of new operational strategies (\$/ton).

ΔE	Energy savings due to ΔP (kW·h/year).
ΔE_j	Total additional energy required to process an additional ton of material output out of the j^{th} process (kW·h/ton).
$\left(\frac{\Delta E}{\Delta \eta}\right)_j$	Energy-to-efficiency sensitivity parameter for j^{th} process (kW·h).
$F_{k,j}$	Boolean variable that indicates whether there is product flow from k^{th} process to j^{th} process.
F_n	Net cash flow in the n^{th} time period ($).
Ft_j	Fixed cost incurred per ton of product arriving at j^{th} process ($/ton).
$\hat{F}t_j$	Increased fixed cost incurred per ton of product arriving at j^{th} process after the introduction of new operational strategies ($/ton).
$i\%$	Real minimum attractive rate of return.
I_a	Expected capital investment for a^{th} new operational strategy ($).
K_j	Maximum processing capacity at j^{th} process (tons/year).
L_j	Material loss percentage for j^{th} process.
MI_j	Total material input for j^{th} process (tons/year).
N	Time horizon for economic analysis (years).
$OS(j,a)$	Boolean variable that indicates whether a^{th} new operational strategy has been implemented in j^{th} process.
$P_{j,k}$	Product quantity transferred from j^{th} process to k^{th} process(tons/year).
$\hat{P}_{j,k}$	Reduced product quantity transferred from j^{th} process to k^{th} process(tons/year).
$\widetilde{P}_j$	Additional amount of product that j^{th} process will receive from its predecessors (tons/year).
PB_j	Power (energy) benefit for j^{th} process (kW·h/year).
PBC_j	Power (energy) benefit coefficient for j^{th} process (kW·h/ton).
$PDCt_j$	Cost that a ton of material that arrives to j^{th} process accumulated as it moved through the preceding processes ($/ton).
PW_j	Total accumulated energy per ton of product arriving at j^{th} process (kW·h/ton)
ΔP	Additional quantity of material resulting from operational improvements (tons/year).
ΔP_j	Total quantity of material recovered at j^{th} process (tons/year).

ΔP_j^A	Total quantity of material loss becoming acceptable-quality product at j^{th} process (tons/year).
ΔP_j^B	Total quantity of material loss becoming recycled product at j^{th} process (tons/year).
ΔP_j^C	Total quantity of recycled product becoming acceptable-quality product at j^{th} process (tons/year).
$\Delta P_{j(a)}$	Quantity of material recovered by a^{th} new operational strategy at j^{th} process (tons/year).
$\Delta P_{j(a)}^A$	Quantity of material loss recovered into acceptable-quality product by a^{th} new operational strategy at j^{th} process (tons/year).
$\Delta P_{j(a)}^B$	Quantity of material loss recovered into recycled product by a^{th} new operational strategy at j^{th} process (tons/year).
$\Delta P_{j(a)}^C$	Quantity of recycled product recovered into acceptable-quality product by a^{th} new operational strategy at j^{th} process (tons/year).
ΔPCC	Process cost savings ($/year).
ΔPDC	Production cost savings ($/year).
$\left(\frac{\Delta P}{\Delta E}\right)_j$	Product-to-energy sensitivity parameter for j^{th} process (ton/kW·h).
R	Enterprise's total profit ($/year).
R^*	Total final profit after the introduction of new operational strategies ($/year)
R_i	Revenue or sales price per ton of i^{th} final product ($/ton).
$r_{j,k,i}$	Ratio of total material output (Type 1 EM model) from or total material input (Type 2 EM model) into j^{th} process either transferred to k^{th} process (represented by $r_{j,k,0}$) or sold as i^{th} final product (represented by $r_{j,0,i}$).
$\hat{r}_{j,k,i}$	Modified value of $r_{j,k,i}$.
$Ti_{(j)}$	Throughput for i^{th} final product coming out of j^{th} process (tons/year).
$T^m{}_i$	Minimum throughput desired for the i^{th} final product (tons/year).
$T^M{}_i$	Maximum throughput desired for the i^{th} final product (tons/year).
ΔT_j	Total additional throughput generated by an additional ton of material output out of j^{th} process (tons/year).
Vt_j	Variable cost incurred per ton of product arriving at j^{th} process ($/ton).

Y_j	Yield for j^{th} process.
$\hat{Y}_j$	Improved yield for j^{th} process.
$\Delta\ \eta$	Percentage of material lost.
$\left(\dfrac{\Delta\eta}{\Delta C}\right)_j$	Efficiency-to-cost sensitivity parameter for j^{th} process ($\$^{-1}$).

INTRODUCTION

Statement of the Problem

The implementation of a new operational strategy (i.e., maintenance policy, new technology, change in process design, etc.) in a production process is one of the major undertakings for any organization. The new operational strategy may have a profound impact on both the physical process, along with its operating characteristics and performance, as well as the financial performance of the entire company.

Traditionally, the worthiness evaluation of a new operational strategy has been done following an "outsider's perspective." In other words, a group of new operational strategies, which are pre-selected using initial cost, required production volume, perceived feasibility or any other overall basis, is "fit" into the existing production process to determine the resulting benefits and costs. Since these new operational strategies are normally selected without consideration for the needs of the current production process, it is very possible for the majority of them to fail to provide significant benefits or to address the most significant constraints of the present system.

Furthermore, the evaluation of the new operational strategies is normally conducted with the objective of optimizing one or some of the elements of the production process and their local measurements. The problem is that while those system elements may experience significant improvements, the financial performance of the organization may be unchanged or degraded given the fact that the new operational strategy fails to achieve a new global optimum.

As a result, organizations find difficult to manage the mysticism surrounding R&D investments, and fail to establish an adequate balance between R&D investments and their related risks due to the significant uncertainty associated with calculating true total process impacts. The fact that the steel industry is highly capital intensive (capital cost represents 20 to 35% of total cost of producing steel) compounds the problem farther.

Importance of Problem

In December 1996, the U.S. Department of Commerce released a report on the competitiveness of the American basic steel industry titled "Meeting the Challenge: U.S. Industry Faces the 21st Century. The Basic Steel Industry" (Cyert and Fruehan, 1996). Although the report acknowledged that the industry had emerged in much better financial and operating condition out of the profound changes it had experienced in the previous 15 years, it was still facing major challenges influencing its competitiveness and ability to grow. These major challenges include:

- *technology* due to the competitive advantages available in the choice of manufacturing technology system,
- *capital economic performance*,
- *government regulations* in the form of rules on discharges and some others that may affect demand for steel,
- *scrap availability, price, and substitutes* due to aggressive and widespread recycling programs, and
- *foreign competition*.

In 1998, the steel industry published the report "Steel Industry Technology Roadmap" describing the technical advances that the steel industry believes are the highest priority if steel is to remain the material of choice in the 21st century (AISI, SMA and DOE, 1998). The report provides an extensive list of R&D needs in four critical areas that an earlier report titled "Steel: A National Resource for the Future" (AISI and SMA, 1995), better known as the American steel industry's vision report, had identified:

- Process efficiency: to seek improvement in throughput, quality and energy efficiency.
- Recycling: to increase steel recycling and recovery from plant solid wastes.
- Environmental engineering: to achieve further reductions in emissions and to develop new processes to avoid pollution rather than control and treat it.
- Product development: to be increasingly responsive to ever-changing market demands.

It is important to note that although the technology roadmap report does an outstanding job translating the vision into a tactical agenda; it did not address any implementation issues, which deal with how a specific steel company can become more profitable and stay at the forefront of manufacturing technology by selecting from the broad inventory of new technologies and strategies that will be developed to satisfy the R&D needs – for example, the report lists 111 R&D needs under the 'process efficiency' area alone, and it is logical to expect that more than one approach will be taken to tackle each need.

At the American Iron and Steel Institute's General Meeting on May 17th, 2000, James Walsh, Executive Vice President of Steel Group, conducted a panel session titled "Technology and the Steel Enterprise." Mr. Walsh indicated that the desire to create a competitive advantage generates the demand for new technology, which in turn is always tied to a capital investment. He also pointed out that "no unregulated industry in recent history has destroyed as much capital as the steel industry," and he suggested to start evaluating technologies through a project and investment analysis.

On September 10th, 2000, Steel Manufacturers Association's president Thomas Danjczek argued during his "Steel Making for the Next Millennium" presentation that U.S. mills need to become more competitive by effecting cost with changes in the operating rates, cost reductions, and technological improvements.

As can be seen, there is a lot of agreement as to what needs to be done, but not much guidance as to how steel organizations can accomplish it. The sustained growth and long-term prosperity of U.S. steel companies will increasingly depend on their ability to quickly assess the true operational and economic impact of new technologies on their businesses. In other words, steel companies need methods to predict and measure how new technologies and strategies will affect plant performance from a global system point of view. Under a Department of Energy project titled "Value Chain Analysis (Value-Derivative) Model for the Steel Industry," the Oak Ridge National Laboratory (ORNL) has developed a prototype model that provides such a methodology.

The proposed value chain analysis (VCA) model, whose overview is presented in the Appendix, "offers a critical tool whereby we define the methodology and data necessary to measure the benefits of an incremental investment in money or effort, to change a technology or operating parameter at any stage of the steel production process, and measure the efficiency, quality, energy savings or any other benefits, at the production stage or at any and all linked process stages throughout the whole plant, and ultimately enable the steel manager to make strategic decisions among quality, value, product mix and evaluate their relative merit per $100 spent" (Dr. Don Barnett, Economic Associates, Inc.).

ORNL has determined that central to the effectiveness of this VCA model is an optimization module that will help determine the best mix of inputs for each process step to maximize the net effect on the performance of the total enterprise. The module needs to measure and respond to changes in individual process outputs, as well as to changes in the mix of process inputs as it relates to the enterprise. The module must also properly represent and accommodate all the internal interconnectivities of the enterprise map.

Contributions

In addition to a new approach and methodology to analyze and optimize the impact of new technologies and operational strategies on multi-stage manufacturing processes, which could obviously be applied across a wide segment of the economy, companies in the steel industry will be able to

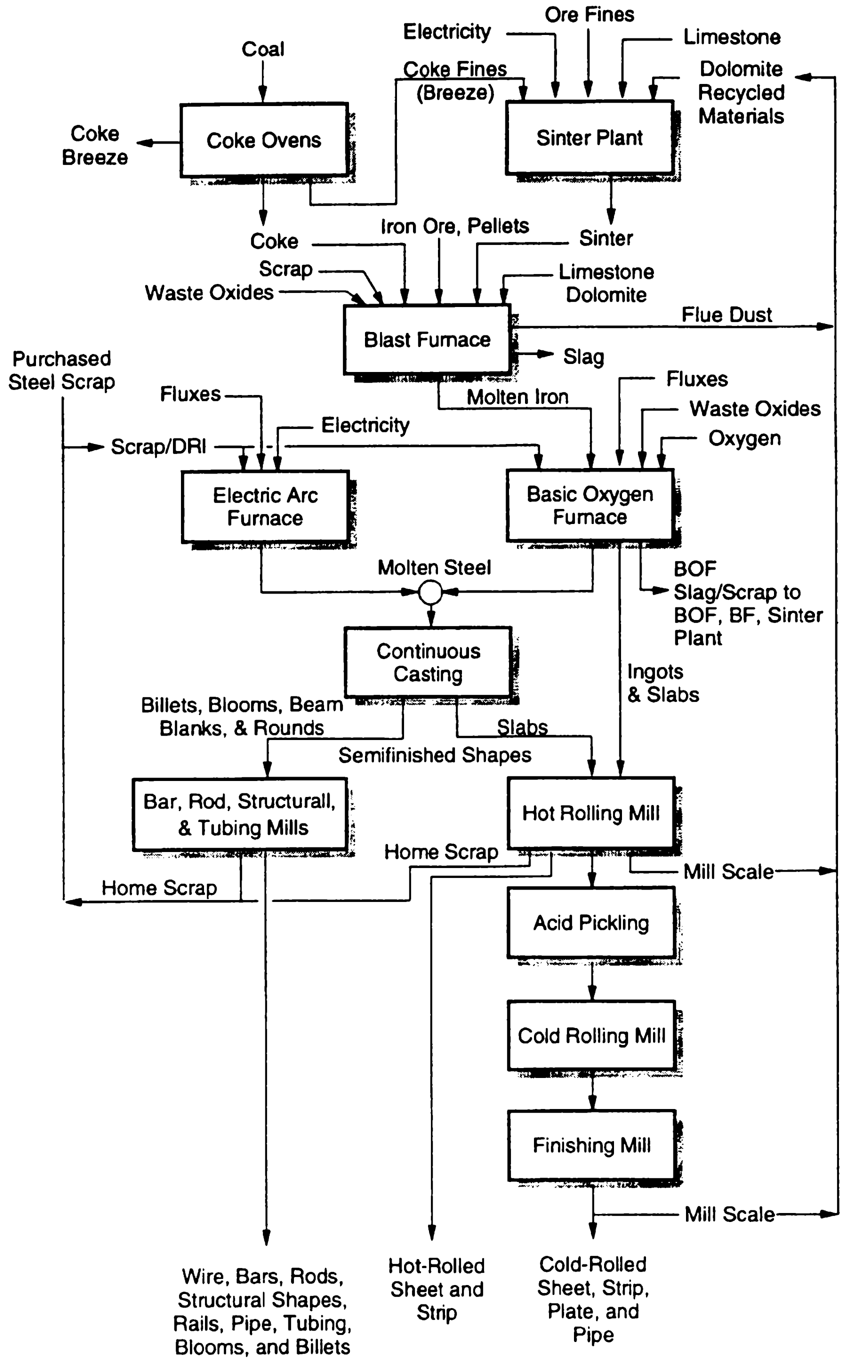
- Map functional needs onto technology requirements,
- Calculate true worth of a technology and its total cost of ownership,
- Select best technology or technologies based on several criteria with a defensible technology selection procedure,
- Identify true cross-cutting technologies,
- Strategize technology investments based on a set of criteria,
- Achieve reduced life-cycle costs through a better understanding of technology impacts on entire process and not just the sub-process, and
- Track technology impact after implementation.

The Steel Manufacturing Process

In order to provide the foundation for the models needed to conduct this research, process flow diagrams for the two current major steel manufacturing process routes are presented in this section: the integrated mill and the mini-mill (AISI, SMA and DOE, 1998). The basic difference between these two process routes comes from the raw materials they use: an integrated mill uses iron ore and coal while a mini-mill utilizes scrap steel. Mini-mills are generally considered leaders in efficiency and ability to implement new technology, and they have gained market share in lower quality products such as bars, structural shapes and flat-rolled steel because of their lower costs and resulting lower prices. Integrated producers, in response, have shifted to producing a wider variety of higher quality, more complex products (Cyert and Fruehan, 1996).

Today, integrated mills produce steel in basic oxygen furnaces (BOF) while mini-mills use the electric arc furnace (EAF) process - see figure 1 for an overview of these two steelmaking processes. In addition, most integrated mills use the blast furnace to produce iron, although direct reduction and iron smelting are gaining popularity. In fact, virtually all iron produced in the U.S. in 1996 came out of blast furnaces, but direct reduction is expected to account for 10 to 15% of the iron produced in the U.S. by 2015. Furthermore, according to the Steel Manufacturers Association (SMA), mini-mills represented 46% of the U.S. steel production and recycled over 70 million tons of ferrous scrap in 1999, and they are anticipated to manufacture 60% of the U.S. steel by 2015.

Figures 2 and 3 show examples of integrated and mini-mill manufacturing processes. The shadowed processes in these figures represent activities with saleable product outputs. Some of these are called semi-finished products such as slabs, blooms and billets out of the continuous caster; and others are finished products such as plates, sheets, strips, wires, rods, bars, beams, tubes and rails out of the finishing mill. Obviously, steel companies have to decide for some of those processes how much of that product output to sell and how much to feed into the subsequent processes.



Source: American Iron and Steel Institute, *Steel Industry Technology Roadmap*, 1998. Adapted from U.S. Council on Wage and Price Stability, *Report to the President on Prices and Costs in the United States Industry*, 1977 (COWPS, October 1977).

Figure 1. Overview of Steelmaking Process

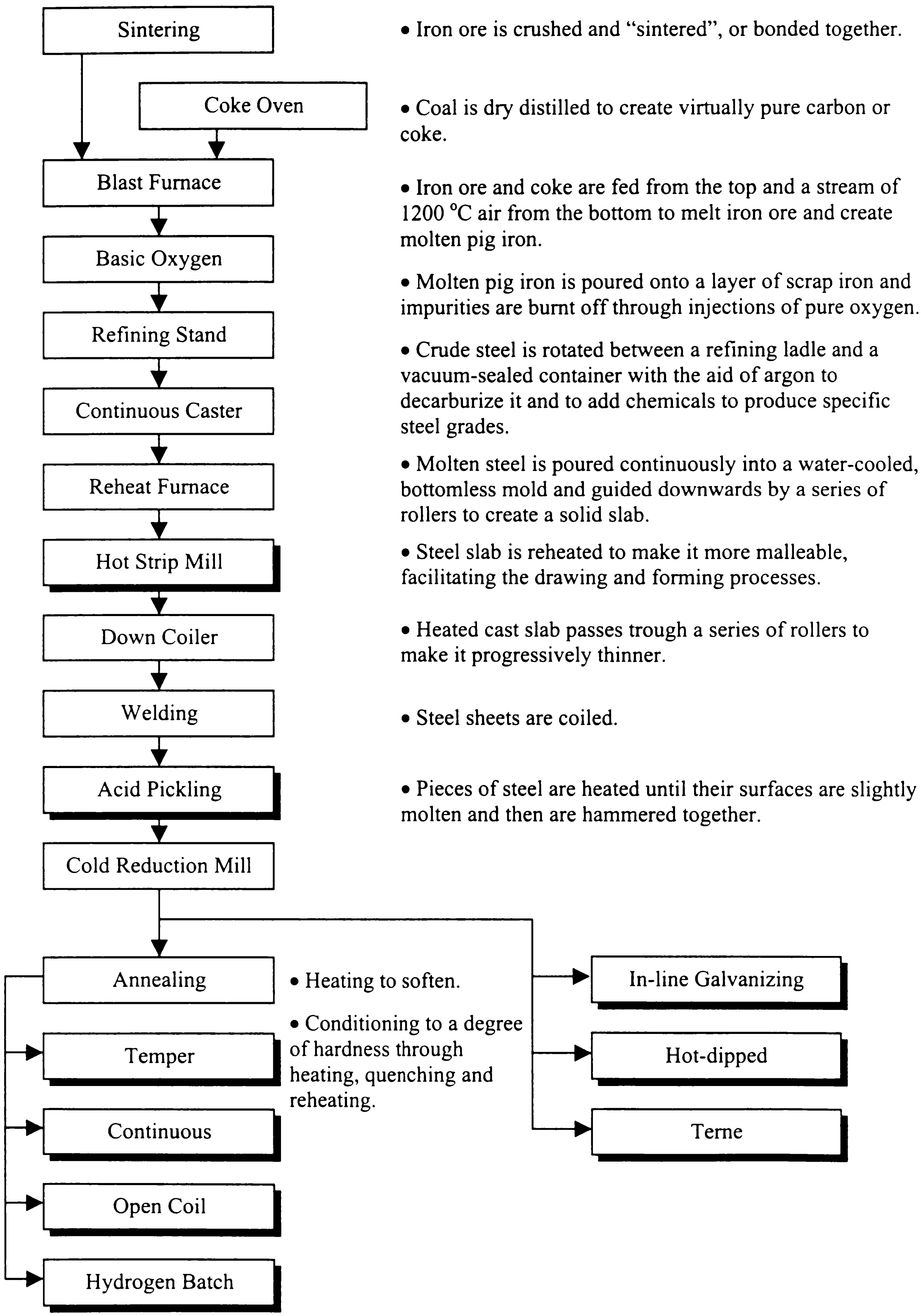
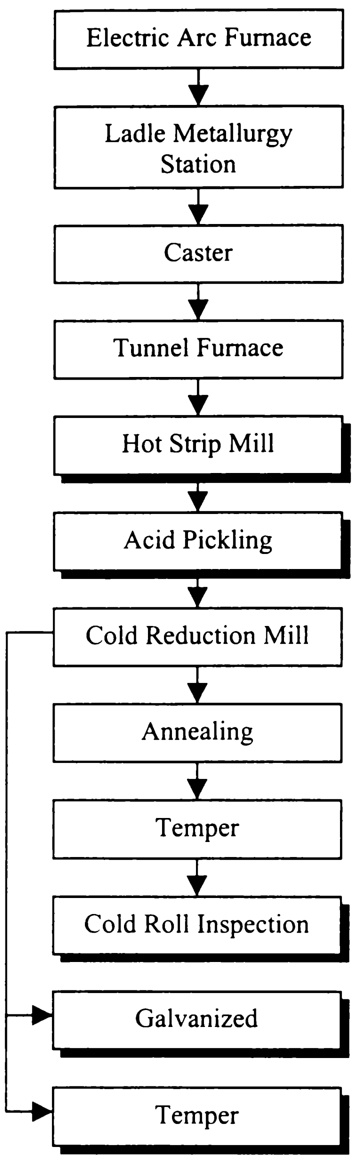


Figure 2. Integrated Mill's Steelmaking Process Flow Diagram



- Untreated or sorted/crushed/calibrated scrap at least 92% iron is melted by powerful electric arcs which "jump" between electrodes.
- Crude molten steel is rotated between a refining ladle and a vacuum-sealed container with the aid of argon to decarburize it and to add chemicals to produce specific steel grades.
- Molten steel is poured continuously into a water-cooled, bottomless mold and guided downwards by a series of rollers to create a solid slab.
- Steel slab is reheated to make it more malleable, facilitating the drawing and forming processes.
- Heated cast slab passes trough a series of rollers to make it progressively thinner.
- Steel is heated to soften.
- Steel is conditioned to a degree of hardness trough heating, quenching and reheating.
- Steel is conditioned to a degree of hardness trough heating, quenching and reheating.

Figure 3. Mini-mill Steelmaking Process Flow Diagram

CHAPTER 1

REVIEW OF THE LITERATURE

This chapter presents a general review of literature on the subjects of capital investment analysis and the application of operation research techniques to the steel industry only. The review of literature for other topics such as model formulation and mathematical programming techniques is included in subsequent chapters in order to make it more relevant and to make the presentation clearer.

1.1 Capital Investment Analysis

Capital investment is a key driver in corporate performance, and the effective allocation of a company's capital resources is a key to corporate success. Consequently, managers must combine the knowledge of the business with sophisticated financial analysis to enhance the probability of making good investment decisions. Gordon and Pinches (1984) propose that a capital investment analysis should include the following activities: strategic analysis, establishing investment goals, searching for investment opportunities, forecasting investment cash flows, risk-adjusted evaluation of forecasted cash flows, decision making, implementation of accepted opportunities, and post-audit operating performance. In fact, Pinches (1982) observes that focusing on the simple selection phase (decision making) is myopic, and that the more global approach is needed to fully understand the capital budgeting process.

The problem of project selection is that of "choosing a compatible set of alternatives that satisfy constraints on capital required, on financial and other resources and on service levels or quality required, and that maximizes some measure of total return" (Tobin, 1999). Lorie and Savage (1955) were the first ones to discuss the problem of selecting independent projects -the selection of one project does not affect the choice of any other as long as the constraints are met- under capital rationing. Since then, two main solution approaches have evolved as follows.

The economic theory method emerged from the fact that selecting projects with the highest economic worth measure, as long as the budget is not exceeded, does not guarantee an optimum allocation even when equivalent worth measures are utilized (Park and Sharp-Bette, 1990). Therefore, it is necessary to define mutually exclusive investment alternatives in terms of combination of operational strategies, and these investment alternatives have to be compared using the following basic philosophy (Sullivan, Bontadelli and Wicks, 2000):

> "*The feasible investment alternative that requires the minimum investment of capital and produces satisfactory functional results will be chosen unless the incremental capital associated with an alternative having a larger investment can be justified with respect to its incremental benefits*".

The basic philosophy can be implemented through either the incremental or the total investment analysis depicted in figure 4.

Weingartner (1963) and Kaplan (1966) proposed the second solution approach when they formulated the Lorie-Savage problem as a binary linear integer program, also known as the 0/1-knapsack problem because it can be interpreted as a problem of selecting a best set of items to go in a hiker's knapsack, given the value he attaches to those items and an upper limit on the amount of weight he can carry (Moder and Elmaghraby, 1978). The 0/1-knapsack problem is a "hard" (NP-complete) combinatorial optimization problem for which no technically sound or efficient (polynomial) algorithms are available (Bjorndal et al., 1995). The difficulty stems primarily from the fact that if q operational strategies are available, there will be 2^q possible feasible solutions and obtaining the solution is expensive and in some cases intractable. Realizing the difficulty of the problem, several researchers have proposed a number of heuristic approaches for solving this type of problem (Wolsey, 1998; Wei, Chena and Tsai, 1999; Karabal, Bean and Lohman, 2000). However, the research has barely extended into the non-linear arena.

It is important to note that a lot of discussion has taken place regarding the form the models should take, especially in relation to the measure of total return to be maximized. Nevertheless, discounted cash flow (DCF) methodologies are considered to be theoretically preferred today because they yield consistent results when the maximization of the wealth is the criterion, and

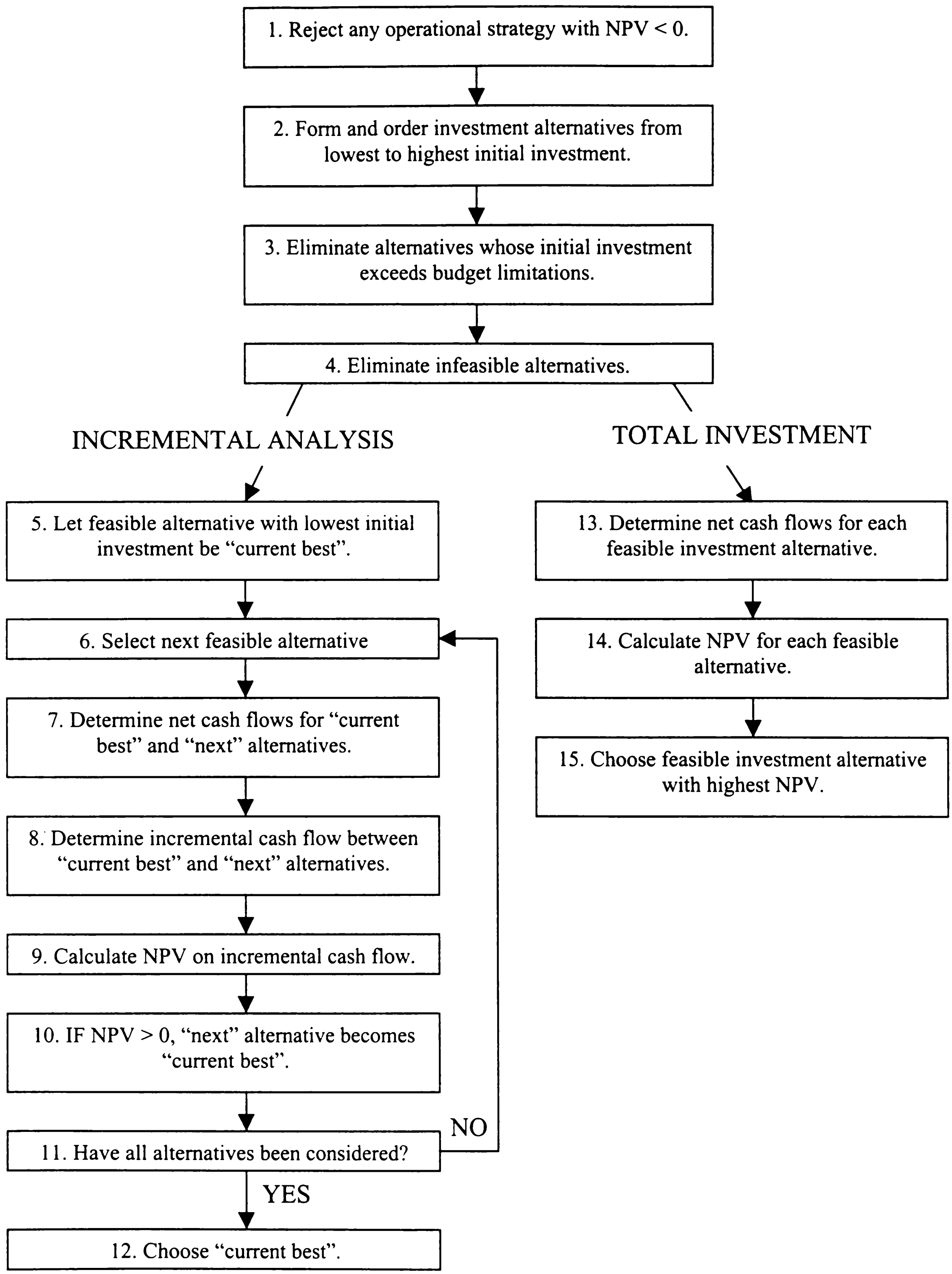


Figure 4. Incremental Analysis and Total Investment Approaches

equivalent worth measures such as net present value (NPV) are regarded as fundamentally correct. Fortunately, according to a number of surveys examining corporate capital investment practices of U.S. companies, the following two general conclusions can be drawn (Farragher, Kleiman and Sahu, 1999):

- DCF measures are the most popular primary evaluation methods, and their usage is increasing over time - in fact, it grew from 57 to 80% between the 1969 and 1999 surveys.
- Few companies employ quantitative risk assessment, and those that do favor sensitivity analysis.

On the other hand, return on investment (ROI) has been regarded as fundamentally incorrect because it is a function of an accounting device for allocating first cost (less salvage value, if any) over the life of the asset. Thus, it is possible to alter apparent profitability by changing the depreciation schedule for the asset in question (Fleischer, 1994). Using the original book value method, ROI is calculated by dividing the average annual accounting profit by the original book value of the asset.

1.2 Operations Research

A number of operations research models have been devoted to the process industries, where a series of engineering operations involving chemical and/or physical change of a substance(s) into some other desired and/or undesired substance(s) are set to achieve a desired final product (Fortuin, van Beek and Wassenhouse, 1996; Yu, 1998). The problems can be classified as:

- Distribution related: the objective is to distribute resources in an optimal fashion. The "customer demand-product source" shipping allocation problem is a good example.
- Blending related: the objective is to blend materials for the purpose of promoting chemical reactions or meeting product specifications. For instance, the chemical reaction yield maximization problem.
- Single operation: for example, the blast furnace production scheduling problem.
- Multiple operation: for instance, the oil refinery scheduling problem.

- Multioperation serial: the objective is to optimize a flow-structured process. The optimum staging of chemical reaction vessels is an example.

Moreover, there have been some attempts to solve the different problems that arise in the steel industry and the solution techniques are quite varied (Lopez, Carter and Gendreau, 1998). The major research areas have included:

- Production Planning: topics such as integrated model for the production planning in a large iron and steel manufacturing environment (Li and Shang, 2001), optimization of steel mill production (Junno, 1989), and continuous caster scheduling optimization (Jawahir, 1998).
- Capacity Analysis: subjects such as capacity analysis for mixed technology production (Couretas et al, 2001).
- Process Control: themes such as real-time stochastic process control system for process manufacturing (Shao, 1993) and dynamic optimization of nonlinear control of chemical processes (Dadebo, 1996).

Clearly, the research has been almost entirely focused on the optimization of the current manufacturing system, while little effort has been devoted to the issue of technology implementation and its impact at the enterprise level.

CHAPTER 2

THE ENTERPRISE MATHEMATICAL MODEL

2.1 Model Formulation

The term *model* is usually used for a structure that has been built purposely to exhibit features and characteristics of some other object or system. This research requires two types of related models: an *enterprise mathematical model* to describe the steel manufacturing process in terms of performance indicators, and a *mathematical programming model* to search for optimal values of the process performance indicators given a set of parameters.

The essential feature of an enterprise mathematical model is that it involves a set of mathematical relationships such as equations, inequalities, logical dependencies, etc., which correspond to some relationships in the real world system. A mathematical programming model, on the other hand, involves optimizing (i.e., maximizing or minimizing) an objective function of n decision variables subject to specified constraints. It should be noted that the relationships in a mathematical model are, to a large extent, independent of the data in the system; while the answer provided by a mathematical programming model is clearly affected by the objective function and the system's data and mathematical model (Williams, 1985).

Even though there are excellent methods for solving a problem once it is formulated, there is little theory to help in formulating problems in a mathematical programming way. Two approaches, however, are commonly used (Murty, 1985; Dantzig and Thapa, 1997): direct and input-output.

The *direct approach*, also known as the *activity approach*, has three steps. The first step is to identify and list all the decision variables in the problem. This list must be complete in the sense that if an optimum solution providing the values of each of the variables is obtained, the decision maker should be able to translate it into an optimum policy that can be implemented. These decision variables may need to meet certain assumptions so that a specific optimization method can be properly applied. For instance, the proportionality assumption guarantees that that if a_{ij} units of the i^{th} item are consumed or produced in carrying out activity j at the unit level, then $a_{ij}x_j$ units of this item are consumed or produced in carrying out activity j at level x_j. The additivity assumption implies that the total consumption or production of an item is equal to the sum of the various quantities of the item consumed or produced in carrying out each individual activity at its specified level. Likewise, the continuity of variation assumption assures that each decision variable can take all the real values in its range of variation, and the nonnegativity assumption ensures that such a range of variation does not include negative values.

The second step of the direct approach deals with the definition of all the constraints in the problem, and the final step involves the determination of the objective function in terms of the decision variables.

The *input-output or material balance approach* starts with a list of all the possible activities and the items defining the constraints. The list of activities should include all the possible actions that the decision maker can perform in the problem; and, consequently, the levels at which the activities are performed define the decision variables in the problem. Moreover, since an item in the problem is any material or resource on which there is either a requirement or a limit on its availability or use, each item leads to a constraint. One or more of the items can be selected to define the objective function(s).

Next, to write the constraints in a tabular form, the approach defines x_j as the level at which the j^{th} activity is carried out, and assigns the j^{th} column in the tableau to correspond to this activity. Likewise, it assigns a row of the tableau to each item. If a_{ij} untis of i^{th} item are required as an input for carrying out the j^{th} activity at unit level, it enters $+a_{ij}$ in the (i,j) position of the tableau. However, if a_{ij} units of the i^{th} item are produced as an output by carrying out the j^{th} activity at unit level, it enters $-a_{ij}$ in the (i,j) position. Entering the limits on the availability or requirements of the items completes the constraints.

2.2 Steel Manufacturing Process Flow Description

The steel manufacturing process has three main distinctive characteristics that must be considered in the models:

1. The material output from any process may be divided to feed a number of subsequent processes.
2. Saleable products, henceforth referred to as *final products*, can come from a final process (a process whose only output is a saleable product) as well as an intermediate process (a process whose part of its output becomes input for another process within the company's manufacturing system).
3. Material feedback loops exist so that a portion of the material output from any process, from now on referred to as *recycled product*, may become material input for the same process or an earlier process.

The steel manufacturing process flow in figure 5 illustrates these characteristics. The material input for the j^{th} process includes both the intermediate product from a preceding process and the recycled product from subsequent process(es), if any. This material input can then become either a material loss or a material output. The material loss consists of process loss and non-recovered recyclable product. In turn, the material output is made up of intermediate product for the next

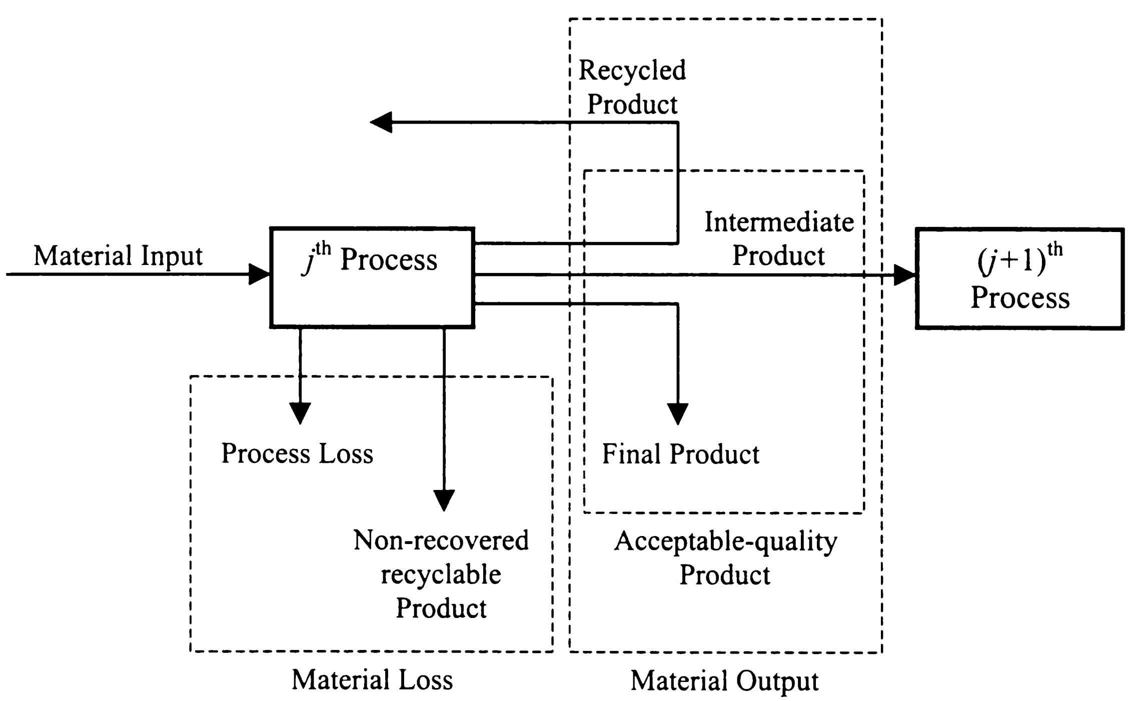


Figure 5. Steel Manufacturing Process Flow

process(es), final product to be sold on the market, and recycled product to be fed back to the same process or an earlier process. Furthermore, the combination of intermediate and final products makes up the acceptable-quality product.

It is important to note that there are four sources of recyclable material (AISI, SMA and DOE, 1998):

- By-products: the residues produced during ironmaking, steelmaking, and rolling operations. These residues include slag, dusts, sludges, and mill scale.
- Home Scrap: the steel scrap generated within steelmaking plants. This category includes leftover pieces of steel from steelmaking, iron and steel recovered from slag processing, and defective or rejected products at the mill.
- Prompt Scrap: the steel scrap generated during the manufacture of steel products. Examples of this type of scrap include punched-out pieces of steel sheet from an appliance manufacturer, turnings from the manufacture of screws and bolts, skeleton scrap from the production of can ends, and side trimming from the manufacture of hoods and bumpers at auto stamping plants.
- Obsolete Scrap: the steel scrap contained in post-consumer products. This category includes such diverse items as discarded cars, appliances, construction materials, containers, other durables, and municipal waste.

Obviously, the recycled product shown in the material feedback loop of figure 5 only includes the portion of by-products and home scrap that are currently being recovered at close to 100 and 50%, respectively (AISI, SMA and DOE, 1998); while the remaining non-recycled portion is represented by the non-recovered recyclable product. Moreover, prompt scrap and obsolete scrap are generated after the final products leave the factory, and they will be assumed to come back in the form of raw material for mini-mills.

2.3 The Enterprise Mathematical Model

The enterprise mathematical (EM) model describes the steel manufacturing process in terms of enterprise performance indicators, which include final product throughput, energy consumption and profit and are represented by the function $f(\mathbf{x})$, where $\mathbf{x}$ is a vector of independent variables.

The EM model will be developed under two different scenarios: without material feedback loops (Type 1) and with them (Type 2). Although the first scenario does not represent the steel manufacturing process as described in the previous section, it will enhance the reader's understanding of the EM model with material feedback loops, which requires the development of a steady-state model as explained later, and it will make this research applicable to other industries where feedback loops do not exist.

Under the Type 1 EM model, it will be assumed that any material loss from a process is scrapped at no cost and any manufacturing cost it incurred is allocated to the remaining acceptable-quality product (Intermediate and final product, if any). Under the Type 2 EM model, any manufacturing cost incurred by the lost material will be allocated to the remaining material output (Intermediate, final and recycled product, if any).

The theoretical formulation for each model is discussed first; and a numerical example, based on the hypothetical production process presented in figure 6, will follow to illustrate and validate the generalized mathematical formulations.

2.3.1 Type 1 EM Model: Without Material Feedback Loops

2.3.1.1 Final Product Throughput

The following variables are used to model the enterprise's final product throughput indicator:

- $Ti_{(j)}$ is the throughput (tons/year) for i^{th} final product coming out of j^{th} process for $i=1,2,\ldots,n$ and $j=1,2,\ldots,m$.
- $P_{j,k}$ is the product quantity (tons/year) transferred from j^{th} process to k^{th} process for $j,k=0,1,2,\ldots,m$; where process 0 represents the raw material supply.
  It is important to note that this variable represents both material output from j^{th} process as well as material input for k^{th} process.
- Y_j is the yield for j^{th} process (no units).

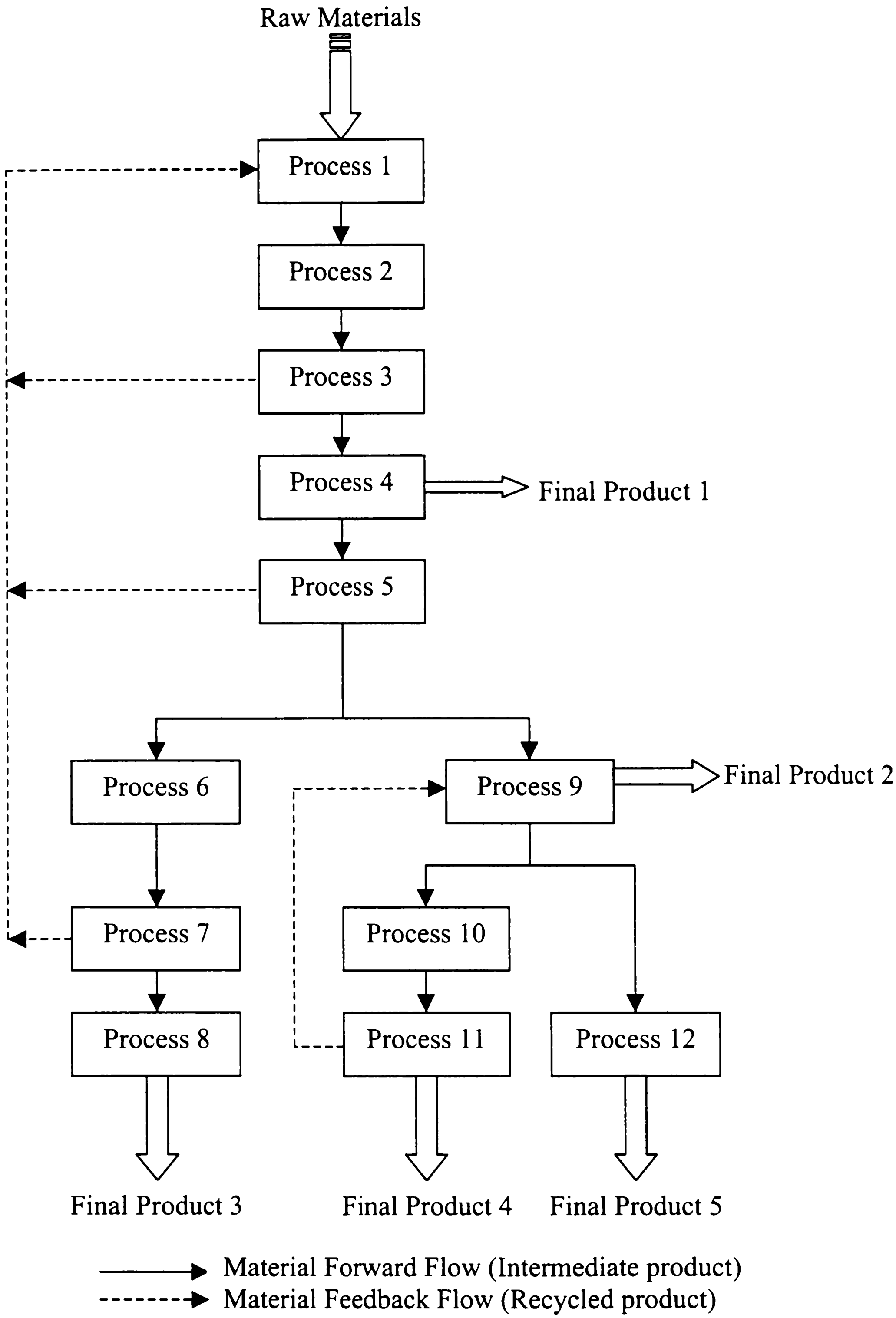


Figure 6. Hypothetical Production Process

- $r_{j,k,i}$ is the percentage (ratio) of total material output from j^{th} process either transferred to k^{th} process (represented by $r_{j,k,0}$) or sold as i^{th} final product (represented by $r_{j,0,i}$) for j,k=0,1,2,…,m and i=1,2,…,n.

The following constraints are placed on the values of $r_{j,k,i}$:

a) $r_{j,k,0} = 0$ for $j \geq k$ to eliminate feedback loops.

b) $r_{0,0,i} = 0$ for i=1,2,…,n to prevent raw material from also being a final product.

c) Only one $r_{0,j,0} \neq 0$ so that only one process receives raw material.

d) $$\sum_{k=1}^{m} r_{j,k,0} + \sum_{i=1}^{n} r_{j,0,i} = 1 \qquad \text{for } j=1,2,\ldots,m \tag{1}$$

to guarantee that exactly 100% of each process' output has a destination downstream.

Eq. (2) and (3) below show how to calculate $P_{j,k}$ and $Ti_{(j)}$ –material outputs from j^{th} process- based on the total material input for j^{th} process (represented by $\sum_{l=0}^{m} P_{l,j}$), and the j^{th} process' yield and distribution percentages

$$P_{j,k} = r_{j,k,0}\ Y_j \sum_{l=0}^{m} P_{l,j} \tag{2}$$

$$Ti_{(j)} = r_{j,0,i}\ Y_j \sum_{l=0}^{m} P_{l,j} \tag{3}$$

Eq. (4) guarantees material balance for each j^{th} process by equating the process's total material input times the process' yield to the process' total material output

$$Y_j \sum_{l=0}^{m} P_{l,j} = \sum_{k=1}^{m} P_{j,k} + \sum_{i=1}^{n} Ti_{(j)} \tag{4}$$

for j = 1,2,…,m; and where only one $P_{l,j}$ is different from zero for each value of j.

2.3.1.2 Energy Consumption

The following variables are used to model the enterprise's energy consumption indicator:

- E is the enterprise's total energy consumption (kW·h/year).
- E_j is the total energy consumed by j^{th} process (kW·h/year).
- Et_j is the energy consumed per ton arriving to j^{th} process (kW·h/year·ton).

The enterprise's total energy consumption can be calculated as the addition of energy consumed by each process, which in turn is equal to the process' total material input times energy consumed per ton that arrives to the process as presented below

$$E = \sum_{j=1}^{m} E_j = \sum_{j=1}^{m} \left(Et_j \sum_{l=0}^{m} P_{l,j} \right) \tag{5}$$

2.3.1.3 Profit

The following variables are used to model the enterprise's profit performance indicator:

- R is the enterprise's total profit (\$/year).
- R_i is the revenue or sales price per ton of i^{th} final product (\$/ton).
- Ft_j is the fixed cost incurred per ton of product arriving at j^{th} process (\$/ton).
- Vt_j is the variable cost incurred per ton of product arriving at j^{th} process (\$/ton).
- C_j is the total accumulated cost per ton of acceptable-quality product after j^{th} process (\$/ton).

The enterprise's total profit is computed as the addition of profit from each final product, which in turn is equal to final product quantity sold -$Ti_{(j)}$- times profit per ton, represented by $\left(R_i - C_j\right)$,

$$R = \sum_{i=1}^{n} (R_i - C_j)\, Ti_{(j)} \tag{6}$$

where only one $Ti_{(j)}$ is different from zero for each value of i,

$$C_j = \frac{\sum_{k=0}^{m}\left(C_k\, F_{k,j}\right) + Ft_j + Vt_j}{Y_j} \tag{7}$$

for $j = 1,2,\ldots m$, and

$$F_{k,j} = \begin{cases} 1 & \text{if } r_{k,j,0} \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

is a Boolean variable that indicates whether there is product flow from k^{th} process to j^{th} process.

A summary of the Type 1 EM model in tableau format is presented in table 1; and a numerical example based on figure 6's hypothetical production process, excluding material feedback flows, is shown in table 2. The model requires input values for the following variables, which are presented in bold font style in the numerical example, to calculate all other variable values:

a) Raw material tonnage represented by $P_{0,j}$.
b) Percentages of product transfers between process and from processes to final products represented by $r_{j,k,i}$.
c) Yield for each process represented by Y_j.
d) Energy consumption per ton for each process represented by Et_j.
e) Fixed and variable cost per ton for each process represented by Ft_j and Vt_j.
f) Cost per ton of raw material represented by C_0, which is equal to $25 in table 2.
g) Revenue per ton of final product represented by R_i.

2.4.1 Type 2 EM Model: With Material Feedback Loops

In the previous section, the material flow was straightforward as illustrated below:

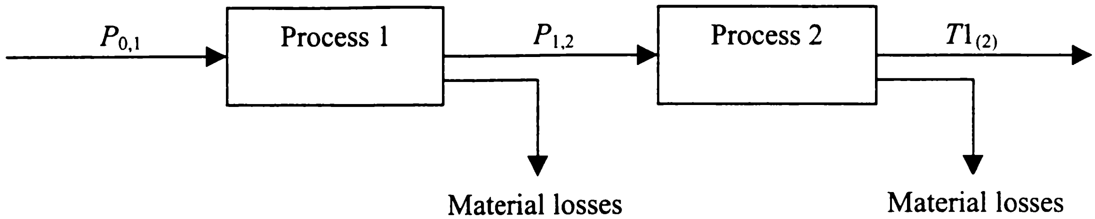
Table 1. Comprehensive Tableau for the Type 1 EM Model

		TO PROCESS				Totals	TO FINAL PRODUCT			Totals
		1	2	…	m		1	..	n	
FROM PROCESS	0	$P_{0,1}$ $r_{0,1,0}$ $F_{0,1}$	$P_{0,2}$ $r_{0,2,0}$ $F_{0,2}$	…	$P_{0,m}$ $r_{0,m,0}$ $F_{0,m}$	$\sum_j P_{0,j}$	$r_{0,0,1}=0$	..	$r_{0,0,n}=0$	$\sum_j r_{0,j,0}+\sum_i r_{0,0,i}=1$
	1	$P_{1,1}=0$ $r_{1,1,0}=0$ $F_{1,1}=0$	$P_{1,2}$ $r_{1,2,0}$ $F_{1,2}$	…	$P_{1,m}$ $r_{1,m,0}$ $F_{1,m}$	$\sum_j P_{1,j}$	$T1_{(1)}$ $r_{1,0,1}$	..	$Tn_{(1)}$ $r_{1,0,n}$	$\sum_i Ti_{(1)}$ $\sum_j r_{1,j,0}+\sum_i r_{1,0,i}=1$
	…	…..	…..	…	…..	…..	…..	..	…..	…..
	m	$P_{m,1}$ $r_{m,1,0}$ $F_{m,1}$	$P_{m,2}$ $r_{m,2,0}$ $F_{m,2}$	…	$P_{m,m}=0$ $r_{m,m,0}=0$ $F_{m,m}=0$	$\sum_j P_{m,j}$	$T1_{(m)}$ $r_{m,0,1}$	..	$Tn_{(m)}$ $r_{m,0,n}$	$\sum_i Ti_{(m)}$ $\sum_j r_{m,j,0}+\sum_i r_{m,0,i}=1$
Totals		$\sum_j P_{j,1}$	$\sum_j P_{j,2}$	…	$\sum_j P_{j,m}$		$\sum_j T1_{(j)}$	..	$\sum_j Tn_{(j)}$	$\sum_i \sum_j Ti_{(j)}$
Yield		Y_1	Y_2	…	Y_m					
Energy/ton		Et_1	Et_2	…	Et_m					
Energy (E_j)		$Et_1\sum_j P_{j,1}$	$Et_2\sum_j P_{j,2}$	…	$Et_m\sum_j P_{j,m}$	$\sum_k\left(Et_k\sum_j P_{j,k}\right)$				
Fixed cost/ton		Ft_1	Ft_2	…	Ft_m					
Var. cost/ton		Vt_1	Vt_2	…	Vt_m					
Acc. cost/ton		C_1	C_2	…	C_m					
Revenue/ton							R_1	..	R_n	
Profit							$(R_1-C_j)\sum_j T1_{(j)}$	..	$(R_n-C_j)\sum_j Tn_{(j)}$	$\sum_i (R_i-C_j)Ti_{(j)}$

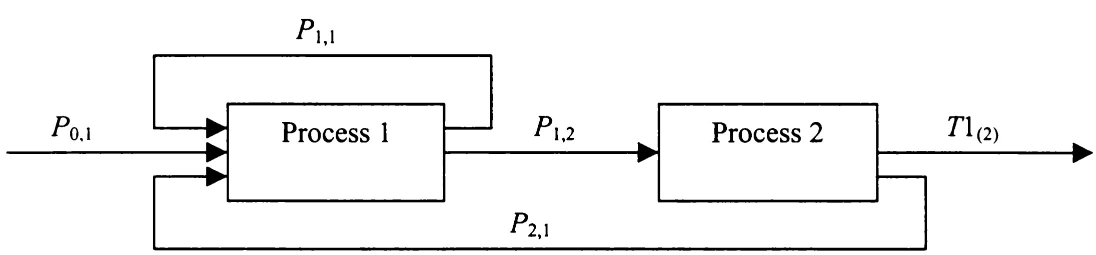
$$C_j=\frac{\sum_{k=0}^{m}(C_k\times F_{k,j})+Ft_j+Vt_j}{Y_j}$$

Table 2. Type 1 EM Model's Numerical Example

			TO PROCESS												Totals		TO FINAL PRODUCT					Totals
			1	2	3	4	5	6	7	8	9	10	11	12			1	2	3	4	5	
FROM PROCESS	0	*P*	10,000	0	0	0	0	0	0	0	0	0	0	0	10,000	*T*						
		r	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	1	0	0	0	0	0	0	0	0	0	0	0								
	1	*P*	0	9,800	0	0	0	0	0	0	0	0	0	0	9,800	*T*	0	0	0	0	0	0
		r	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	0	1	0	0	0	0	0	0	0	0	0	0								
	2	*P*	0	0	9,702	0	0	0	0	0	0	0	0	0	9,702	*T*	0	0	0	0	0	0
		r	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	0	0	1	0	0	0	0	0	0	0	0	0								
	3	*P*	0	0	0	9,508	0	0	0	0	0	0	0	0	9,508	*T*	0	0	0	0	0	0
		r	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	0	0	0	1	0	0	0	0	0	0	0	0								
	4	*P*	0	0	0	0	8,300	0	0	0	0	0	0	0	8,300	*T*	922	0	0	0	0	922
		r	0.00	0.00	0.00	0.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.10	0.00	0.00	0.00	0.00	1.00
		F	0	0	0	0	1	0	0	0	0	0	0	0								
	5	*P*	0	0	0	0	0	2,391	0	0	5,578	0	0	0	7,968	*T*	0	0	0	0	0	0
		r	0.00	0.00	0.00	0.00	0.00	0.30	0.00	0.00	0.70	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	0	0	0	0	0	1	0	0	1	0	0	0								
	6	*P*	0	0	0	0	0	0	2,223	0	0	0	0	0	2,223	*T*	0	0	0	0	0	0
		r	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	0	0	0	0	0	0	1	0	0	0	0	0								
	7	*P*	0	0	0	0	0	0	0	2,045	0	0	0	0	2,045	*T*	0	0	0	0	0	0
		r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	0	0	0	0	0	0	0	1	0	0	0	0								
	8	*P*	0	0	0	0	0	0	0	0	0	0	0	0	0	*T*	0	0	2,025	0	0	2,025
		r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	1.00	0.00	0.00	1.00
		F	0	0	0	0	0	0	0	0	0	0	0	0								
	9	*P*	0	0	0	0	0	0	0	0	0	2,705	0	1,623	4,328	*T*	0	1,082	0	0	0	1,082
		r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00	0.30		*r*	0.00	0.20	0.00	0.00	0.00	1.00
		F	0	0	0	0	0	0	0	0	0	1	0	1								
	10	*P*	0	0	0	0	0	0	0	0	0	0	2,570	0	2,570	*T*	0	0	0	0	0	0
		r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	0	0	0	0	0	0	0	0	0	0	1	0								
	11	*P*	0	0	0	0	0	0	0	0	0	0	0	0	0	*T*	0	0	0	2,416	0	2,416
		r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	1.00	0.00	1.00
		F	0	0	0	0	0	0	0	0	0	0	0	0								
	12	*P*	0	0	0	0	0	0	0	0	0	0	0	0	0	*T*	0	0	0	0	1,477	1,477
		r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	1.00	1.00
		F	0	0	0	0	0	0	0	0	0	0	0	0								
P Totals			10,000	9,800	9,702	9,508	8,300	2,391	2,223	2,045	5,578	2,705	2,570	1,623		*T*	922	1,082	2,025	2,416	1,477	7,922
Yield			0.980	0.990	0.980	0.970	0.960	0.930	0.920	0.990	0.970	0.950	0.940	0.910								
Energy/ton			20	16	22	18	22	15	21	19	21	17	21	16								
Energy			200,000	156,800	213,444	171,143	182,610	35,858	46,687	38,861	117,136	45,990	53,970	25,971	1,288,470							
Fixed cost/ton			7.0	10.0	10.0	4.0	8.0	4.0	5.0	5.0	3.0	6.0	9.0	4.0								
Var. cost/ton			5.0	8.0	4.0	8.0	6.0	7.0	3.0	2.0	5.0	3.0	2.0	4.0								
Acc. cost/ton			37.8	56.3	71.8	86.3	104.5	124.2	143.7	152.2	116.0	131.6	151.7	136.3								
Revenue/ton																	100.0	135.0	165.0	175.0	145.0	
Profit																	12,595	20,555	25,838	56,325	12,896	128,209



If material feedback loops are added, as presented below, and we assume *no material losses* for the purpose of this illustration; the values of $P_{1,1}$, $P_{1,2}$, $P_{2,1}$ and $T1_{(2)}$ will change over time and will asymptotically approach their steady state's values as exemplified next. If the production process is empty at time 0, Y_1 and Y_2 are equal to 0.9 and 0.8, respectively, and $P_{0,1}$ is 100 tons; the system will produce 72 tons of $T1_{(2)}$ final product by the end of the first production cycle.



During the second production cycle, process 1 will have a total material input equivalent to 128 tons -100, 10 and 18 tons from $P_{0,1}$, $P_{1,1}$ and $P_{2,1}$, respectively-; and the system will produce 92.16 tons of $T1_{(2)}$ final product, while 12.80 and 23.04 tons will be sent back to processes 1 and 2, respectively. In other words, process 1's total material input for the third production cycle has grown to 135.84 tons. If this process is repeated a number of times, process 1's total material input ($P_{0,1}$, $P_{1,1}$ and $P_{2,1}$ combined) will approach 138.888 tons, with $P_{1,1}$ and $P_{2,1}$ equal to 13.888 and 25.000 tons, respectively.

As a matter of fact, the system will also asymptotically approach product cost figures for the steady state, even after stability for product flow has been achieved, as demonstrated next. If we assume that the purchase cost is \$10/ton, and Vt_1, Ft_1, Vt_2 and Ft_2 are equal to \$3, \$3, \$2 and \$2/ton, respectively; then

$C_1 = 10 + 3 + 3 = \$16$ (applies to each ton of $P_{1,1}$ and $P_{1,2}$)

$C_2 = 16 + 2 + 2 = \$20$ (applies to each ton of $P_{2,1}$ and $T1_{(2)}$)

after the first iteration.

In the second iteration, these values are calculated as

$$C_1 = \frac{100 \times \$10 + 13.888 \times \$16 + 25 \times \$20 + 138.888 \times (\$3 + \$3)}{138.888} = \$18.40$$

where the first three terms in the numerator represent the cost of feeding 138.888 tons of material into process 1 in the second iteration, and the last term in the numerator corresponds to the fixed and variable costs this total material input will incur as it goes through the first process; and

$$C_2 = \$18.40 + \$2 + \$2 = \$22.40 .$$

In the third iteration,

$$C_1 = \frac{100 \times \$10 + 13.888 \times \$18.40 + 18 \times \$22.40 + 138.888 \times (\$3 + \$3)}{138.888} = \$19.072$$

$$C_2 = \$19.072 + \$2 + \$2 = \$23.072 .$$

If this process is repeated a number of times, C_1 and C_2 will approach \$19.333 and \$23.333, respectively. Obviously, as the number of processes and material feedback loops increase, so do the number of calculations for the entire system. Therefore, it is necessary to generate the equations for the steady state of the Type 2 EM model, hereafter referred to as the *steady-state model.*

2.4.1.1 Steady-State Model

A steady state is present when a system ceases to change any of its state variables implying that all mass and energy transfers at the system boundary are invariant with time (Sonntag, Borgnakke and Van Wylen, 1998). In other words, there is no accumulation or depletion of material within the system boundaries. For instance, in the example given in the previous section, the system's material input and output are equal to 100 tons once the steady state is attained. Furthermore, 38.888 tons always remain within the system and the costs per ton of material out of processes 1 and 2 become constant over time.

2.4.1.2 Final Product Throughput

When material feedback loops are added to the EM model, we need the variables to be able to describe recycled product quantity, if any, from each process. Consequently:

- $Ti_{(j)}$ is the throughput (tons/year) for i^{th} final product coming out of j^{th} process for i=1,2,…,n and j=1,2,…,m.
- $P_{j,k}$ is the product quantity (tons/year) transferred from j^{th} process to k^{th} process for j,k=0,1,2,…,m.
  In fact, if $j < k$, $P_{j,k}$ represents an intermediate product. But if $j \geq k$, $P_{j,k}$ represents a recycled product.
- Y_j is the yield for j^{th} process (no units), calculated as the acceptable quality product (intermediate and final) divided by the total material input,

$$Y_j = \frac{\sum_{k=j+1}^{m} P_{j,k} + \sum_{i=1}^{n} Ti_{(j)}}{\sum_{l=0}^{m} P_{l,j}} \tag{9}$$

- $r_{j,k,i}$ is the percentage (ratio) of total material input to j^{th} process either transferred to k^{th} process (represented by $r_{j,k,0}$) or sold as i^{th} final product (represented by $r_{j,0,i}$) for j,k=0,1,2,…,m and i=1,2,…,n.

The following constraints are placed on the values of $r_{j,k,i}$:

a) $r_{0,0,i} = 0$ for i=1,2,…,n to prevent raw material from also being a final product.

b) Only one $r_{0,j,0} \neq 0$ so that only one process receives raw material.

c) Only one $r_{j,k,0} \neq 0$ for each value of j where $j \geq k$ so that recycled product from j^{th} process only goes to one preceding process.

d) $\sum_{k=1}^{m} r_{j,k,0} + \sum_{i=1}^{n} r_{j,0,i} \leq 1$ for j=1,2,…,m (10)

since at most 100% of each process' material input can become material output (acceptable-quality and recycled product), and material losses do not have $r_{j,k,i}$ values assigned to them.

Eq. (11) shows the relationship between yield and material input distribution percentages,

$$Y_j = \sum_{k=j+1}^{m} r_{j,k,0} + \sum_{i=1}^{n} r_{j,0,i} \tag{11}$$

for $j = 1,2,\ldots,m$.

In turn, Eq. (12) and (13) below show how to calculate $P_{j,k}$ and $Ti_{(j)}$ –material outputs from j^{th} process- based on the total material input for j^{th} process (represented by $\sum_{l=0}^{m} P_{l,j}$) and its distribution percentages

$$P_{j,k} = r_{j,k,0} \sum_{l=0}^{m} P_{l,j} \tag{12}$$

$$Ti_{(j)} = r_{j,0,i} \sum_{l=0}^{m} P_{l,j} \tag{13}$$

However, Eq. (12) and (13) are useful only once material feedback flow quantities, which may constitute part of the total material input j^{th} process and are represented by $\sum_{l=j}^{m} P_{l,j}$, are known. In other words, we need a methodology to calculate material flows under a steady-state model.

A first approach is to generate equations whose solution provides the material feedback flow quantity. For instance, for the single-loop system in figure 7, the following equation can be constructed and solved for $P_{j+3,j}$,

$$P_{j+3,j} = (P_{j-1,j} + P_{j+3,j})\, r_{j,j+1,0}\, r_{j+1,j+2,0}\, r_{j+2,j+3,0}\, r_{j+3,j,0}$$

However, for the nested-loop system in figure 7, this approach requires solving the following system of equations,

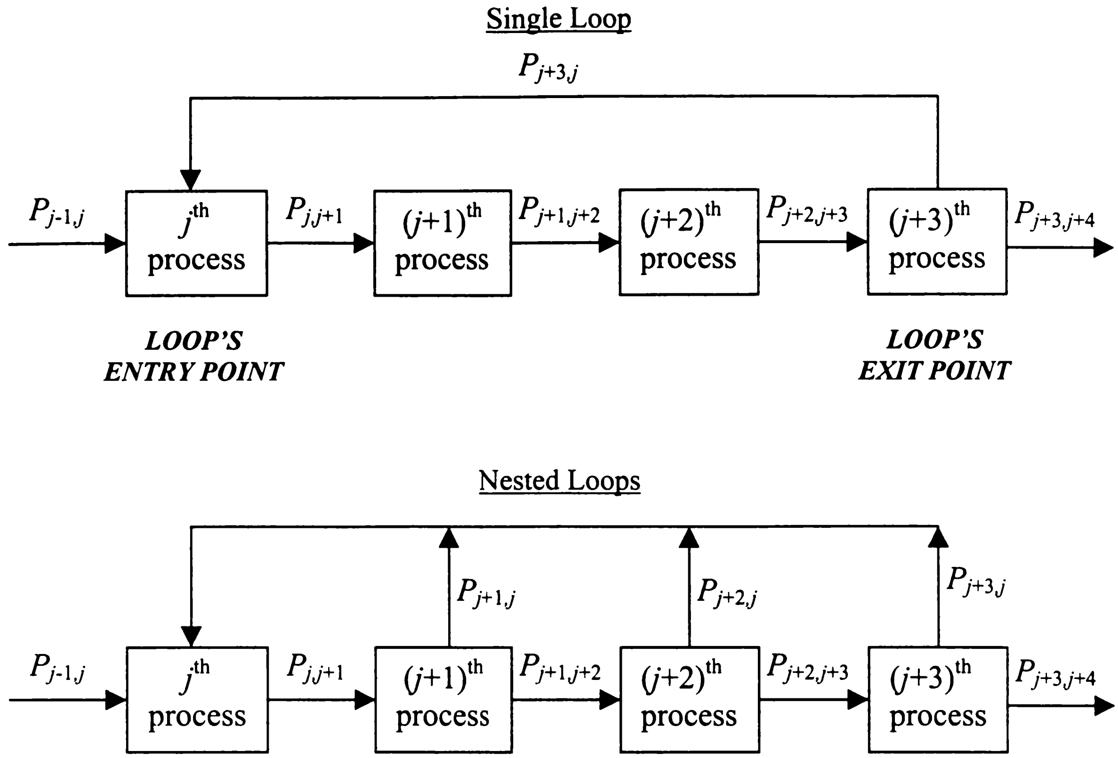


Figure 7. Types of Material Feedback Loops

$$P_{j+3,\,j} = (P_{j-1,\,j} + P_{j+1,\,j} + P_{j+2,\,j} + P_{j+3,\,j})\; r_{j,\,j+1,0}\, r_{j+1,\,j+2,0}\, r_{j+2,\,j+3,0}\, r_{j+3,\,j,0}$$

$$P_{j+2,\,j} = (P_{j-1,\,j} + P_{j+1,\,j} + P_{j+2,\,j} + P_{j+3,\,j})\; r_{j,\,j+1,0}\, r_{j+1,\,j+2,0}\, r_{j+2,\,j,0}$$

$$P_{j+1,\,j} = (P_{j-1,\,j} + P_{j+1,\,j} + P_{j+2,\,j} + P_{j+3,\,j})\; r_{j+1,\,j,0}$$

A second approach generates a single equation, regardless of the number of loops, whose solution provides the total material input. This equation is based on the fact that, in a steady-state system, the material input must equal the material output through the system's boundary. As a result, the equation must be built as follows:

1. Single loop: the equation covers the only loop.
2. Nested loops: when multiple loops exist with a common entry point, the equation covers the one loop that defines the entry and exit points of the combined loop.

In order to simplify the construction of the equation, two variables will be used as follows:

- MI_j is the total material input for j^{th} process (tons/year), and it is calculated as

$$MI_j = \sum_{l=0}^{m} P_{l,j} \qquad \text{for } j=1,2,\ldots,m \qquad (14)$$

- L_j is the material loss percentage for j^{th} process, and it is calculated as

$$L_j = 1 - \sum_{k=1}^{m} r_{j,k,0} - \sum_{i=1}^{n} r_{j,0,i} \qquad \text{for } j=1,2,\ldots,m \qquad (15)$$

If we use the nested-loop system in figure 7, the following equation, which makes the intermediate product coming into the loop equal to the addition of the material losses within the loop and the acceptable-quality product at the end of the loop to maintain material balance, results

$$\begin{aligned} P_{j-1,j} = {} & L_j MI_j + r_{j,j+1,0} L_{j+1} MI_j + r_{j,j+1,0} r_{j+1,j+2,0} L_{j+2} MI_j \\ & + r_{j,j+1,0} r_{j+1,j+2,0} r_{j+2,j+3,0} L_{j+3} MI_j + r_{j,j+1,0} r_{j+1,j+2,0} r_{j+2,j+3,0} r_{j+3,j+4,0} MI_j \end{aligned} \qquad (16)$$

which is easily solved for MI_j.

It is important to note that if any final product is created within the loop, its material output has to be included in the equation. For example, if final product $Ti_{(j+2)}$ is added to the nested-loop system in figure 7, the term $\left(r_{j,j+1,0}\ r_{j+1,j+2,0}\ r_{j+2,0,i}\ MI_j\right)$ must be added to the right side of Eq. (16).

2.4.1.3 Energy Consumption

Same equation as for Type 1 EM model.

2.4.1.4 Profit

Although Eq. (6) remains valid to calculate the enterprise's total profit under the Type 2 EM model, the introduction of material feedback loops affects the allocation of costs among the different product flows due to the intrinsic cost that any recycled product carries upstream. In order to properly calculate product cost, processes have to be classified in two groups:

1. Independent: those processes that are not within the entry and exit points of any loop.
2. Dependent: those processes that are part of a loop.

If a process is independent, Eq. (7) will still be used to calculate the accumulated cost of its material output. However, to calculate accumulated costs for a set of dependent processes, we will rely on the fact that, in a steady-state system, the cost input must equal the cost output through the system's boundary. Cost input refers to the cost of the intermediate product entering the loop plus the combined processing (fixed and variable) costs within the loop, while cost output applies to the cost of the loop's combined material output.

Consequently, if a loop includes processes from j (entry process) to $j+h$ (exit process) and we assume that only the $(j+h)^{th}$ process generates material output (as in figure 7's loops), the following equation is used to calculate C_{j+h},

$$C_{j-1} P_{j-1,j} + \sum_{k=j}^{j+h} \left(\left[Ft_k + Vt_k \right] \sum_{l=0}^{m} P_{l,k} \right) = C_{j+h} \left(\sum_{k=0}^{m} P_{j+h,k} + \sum_{i=1}^{n} Ti_{(j+h)} \right) \tag{17}$$

where the left side of Eq. (17) is the cost input and the right side is the cost output. Then, to calculate the accumulated costs for the rest of the processes in the loop, we need to work our way back one process at a time from the exit point to the entry point of the loop given that we already know C_{j+h}. Eq. (18) is used to calculate C_k if C_{k+1} is known,

$$C_k = \left(1 - L_{k+1}\right) C_{k+1} - Ft_{k+1} - Vt_{k+1} \tag{18}$$

If a given k^{th} process within the loop, $j \leq k < j+h$, generates final product or transfers intermediate product out of the loop, the term $\left(C_k Ti_{(k)}\right)$ or $\left(C_k P_{k,k+l}\right)$ must be added to the right side of Eq. (17), respectively, since it represents a cost output. Furthermore, Eq. (19) must be used to express C_k as a linear combination of C_{j+h} to solve Eq. (17),

$$C_k = \left(\prod_{l=k+1}^{j+h} (1 - L_j) \right) C_{j+h} - \left(\prod_{l=k+1}^{j+h} (1 - L_j) \right) \frac{S_{j+h-1} + Ft_{j+h} + Vt_{j+h}}{1 - L_{j+h}} \tag{19}$$

where S_{j+h-1} is a term of the series defined by

$$S_k = 0 \qquad \text{and} \qquad S_l = \frac{S_{l-1} + Ft_l + Vt_l}{1 - L_j}$$

A numerical example of the Type 2 EM model based on figure 6's hypothetical production process is presented in table 3. The model requires input values for the following variables, which are shown in bold font style in the numerical example, to calculate all other variable values:

a) Raw material tonnage represented by $P_{0,j}$.
b) Percentages of product transfers between process and from processes to final products represented by $r_{j,k,i}$.
c) Energy consumption per ton for each process represented by Et_j.
d) Fixed and variable cost per ton for each process represented by Ft_j and Vt_j.
e) Cost per ton of raw material represented by C_0, which is equal to \$25 in table 3.
f) Revenue per ton of final product represented by R_i.

The last two columns in table 3 show L_j values for all processes as calculated using Eq. (15) and MI_j values for processes 1 and 9 –the loops' entry points- as calculated using an equation similar to Eq. (16). Moreover, the four recycled product flows are highlighted with a gray background.

Table 3. Type 2 EM Model's Numerical Example

		TO PROCESS 1	2	3	4	5	6	7	8	9	10	11	12	Totals		TO FINAL PRODUCT 1	2	3	4	5	Totals	Material loss %	Material input
FROM PROCESS 0	P	10,000	0	0	0	0	0	0	0	0	0	0	0	10,000	T								
	r	1.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000		r	0.000	0.000	0.000	0.000	0.000	1.000	0.000	
	F	1	0	0	0	0	0	0	0	0	0	0	0										
1	P		10,577	0	0	0	0	0	0	0	0	0	0	10,577	T	0	0	0	0	0	0		
	r	0.000	0.980	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000		r	0.000	0.000	0.000	0.000	0.000	0.980	0.020	10,792
	F	0	1	0	0	0	0	0	0	0	0	0	0										
2	P	0	0	10,471	0	0	0	0	0	0	0	0	0	10,471	T	0	0	0	0	0	0		
	r	0.000	0.000	0.990	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000		r	0.000	0.000	0.000	0.000	0.000	0.990	0.010	
	F	0	0	1	0	0	0	0	0	0	0	0	0										
3	P	314	0	0	9,947	0	0	0	0	0	0	0	0	10,261	T	0	0	0	0	0	0		
	r	0.030	0.000	0.000	0.950	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000		r	0.000	0.000	0.000	0.000	0.000	0.980	0.020	
	F	1	0	0	1	0	0	0	0	0	0	0	0										
4	P	0	0	0	0	8,684	0	0	0	0	0	0	0	8,684	T	965	0	0	0	0	965		
	r	0.000	0.000	0.000	0.000	0.873	0.000	0.000	0.000	0.000	0.000	0.000	0.000		r	0.097	0.000	0.000	0.000	0.000	0.970	0.030	
	F	0	0	0	0	1	0	0	0	0	0	0	0										
5	P	434	0	0	0	0	2,371	0	0	5,532	0	0	0	8,337	T	0	0	0	0	0	0		
	r	0.050	0.000	0.000	0.000	0.000	0.273	0.000	0.000	0.637	0.000	0.000	0.000		r	0.000	0.000	0.000	0.000	0.000	0.960	0.040	
	F	1	0	0	0	0	1	0	0	1	0	0	0										
6	P	0	0	0	0	0	0	2,205	0	0	0	0	0	2,205	T	0	0	0	0	0	0		
	r	0.000	0.000	0.000	0.000	0.000	0.000	0.930	0.000	0.000	0.000	0.000	0.000		r	0.000	0.000	0.000	0.000	0.000	0.930	0.070	
	F	0	0	0	0	0	0	1	0	0	0	0	0										
7	P	44	0	0	0	0	0	0	1,984	0	0	0	0	2,028	T	0	0	0	0	0	0		
	r	0.020	0.000	0.000	0.000	0.000	0.000	0.000	0.900	0.000	0.000	0.000	0.000		r	0.000	0.000	0.000	0.000	0.000	0.920	0.080	
	F	1	0	0	0	0	0	0	1	0	0	0	0										
8	P	0	0	0	0	0	0	0	0	0	0	0	0	0	T	0	0	1,964	0	0	1,964		
	r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000		r	0.000	0.000	0.990	0.000	0.000	0.990	0.010	
	F	0	0	0	0	0	0	0	0	0	0	0	0										
9	P	0	0	0	0	0	0	0	0	0	2,708	0	1,625	4,333	T	0	1,083	0	0	0	1,083		
	r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.485	0.000	0.291		r	0.000	0.194	0.000	0.000	0.000	0.970	0.030	5,583
	F	0	0	0	0	0	0	0	0	0	1	0	1										
10	P	0	0	0	0	0	0	0	0	0	0	2,572	0	2,572	T	0	0	0	0	0	0		
	r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.950	0.000		r	0.000	0.000	0.000	0.000	0.000	0.950	0.050	
	F	0	0	0	0	0	0	0	0	0	0	1	0										
11	P	0	0	0	0	0	0	0	0	51	0	0	0	51	T	0	0	0	2,367	0	2,367		
	r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.020	0.000	0.000	0.000		r	0.000	0.000	0.000	0.920	0.000	0.940	0.060	
	F	0	0	0	0	0	0	0	0	1	0	0	0										
12	P	0	0	0	0	0	0	0	0	0	0	0	0	0	T	0	0	0	0	1,478	1,478		
	r	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000		r	0.000	0.000	0.000	0.000	0.910	0.910	0.090	
	F	0	0	0	0	0	0	0	0	0	0	0	0										
P Totals		10,792	10,577	10,471	9,947	8,684	2,371	2,205	1,984	5,583	2,708	2,572	1,625		T	965	1,083	1,964	2,367	1,478	7,858		
Yield		0.980	0.990	0.950	0.970	0.910	0.930	0.900	0.990	0.970	0.950	0.920	0.910										
Energy/ton		20	16	22	18	22	15	21	19	21	17	21	16										
Energy		215,848	169,225	230,358	179,051	191,047	35,561	46,300	37,702	117,246	46,033	54,021	25,995	1,348,386									
Fixed cost/ton		7.0	10.0	10.0	4.0	8.0	4.0	5.0	5.0	3.0	6.0	9.0	4.0										
Var. cost/ton		5.0	8.0	4.0	8.0	6.0	7.0	3.0	2.0	5.0	3.0	2.0	4.0										
Acc. cost/ton		43.4	62.0	77.5	92.3	110.7	130.9	151.0	159.6	122.9	138.8	159.4	143.8										
Revenue/ton																100.0	135.0	165.0	175.0	145.0			
Profit																7,430	13,130	10,678	36,963	1,742	69,943		

CHAPTER 3

THE ENTERPRISE MATHEMATICAL PROGRAMMING MODEL

3.1 Optimization Approaches

Four different optimization approaches can be used by the VCA's model optimization module (see Appendix) as follows:

1) Risk Analysis

Risk analysis determines the process and economic impact of a single new operational strategy.

If only one operational strategy is to be analyzed, the process and economic impact analysis becomes a straightforward activity using the coupled equations of the EM model if the product mix ratios are maintained. However, the optimization module can also provide a sensitivity analysis for $f(\mathbf{x})$ based on per-unit-changes of $Ti_{(j)}$. If more than one operational strategy is to be analyzed concurrently, the optimization module can use the EM model's coupled equations to estimate changes in $f(\mathbf{x})$ assuming the same product mix ratios. As before, the module can also provide the sensitivity analysis figures for each operational strategy, and ultimately suggest the best implementation sequence for the given operational strategies.

2) Alternative Analysis

Alternative analysis deals with contrasting and comparing two or more operational strategies.

The optimization module can calculate changes in $f(\mathbf{x})$ for each operational strategy using the coupled equations in the EM model, and provide a ranking based on production cost savings, additional throughput and energy savings assuming the product mix ratios are not changed. As before, the module can also provide the sensitivity analysis figures for each operational strategy.

3) Enterprise Optimization

Enterprise optimization selects the operational strategy or group of operational strategies that will maximize or minimize an objective function given a set of criteria.

4) Multiplant Optimization

Multiplant optimization is used when an organization has more than one steel-manufacturing facility.

This research provides the foundation to implement the first three approaches. In fact, the main topic of this chapter is *Enterprise Optimization* through the formulation of enterprise mathematical programming (EMP) models, which search for optimal values of the enterprise performance indicators included in the EM model based on a given objective function and a set of constraints.

EMP models will be formulated so that enterprise optimization can be performed under four different philosophies:

1) Increase throughput and maximize profit.
2) Maintain throughput and maximize profit.
3) Increase throughput and minimize the marginal energy consumption increase.
4) Maintain throughput and minimize total energy consumption.

Since the EMP models will be based on the sensitivity parameter concept of the VCA model, it is important to clarify their economic representation. To do this, we must first define the ways in which a new operational strategy can improve a process.

The first is through directly effecting operating parameters. For instance, if Et represents the energy required to process a ton of product that arrives at a process A, and P represents the quantity of product to be processed so that $P \times Et$ is the total energy consumption at process A; a new operational strategy that lowers the value of Et to Et_f will obviously yield an improvement due to a total energy consumption reduction equivalent to $P(Et-Et_f)$.

The second way is through enhanced utilization of the process' existing resources. For example, if Y represents the yield of process A, we need to use Et/Y units of energy to get a ton of acceptable-quality product out of process A since (1-Y)% of incoming tons will become waste or below-quality output. If a new operational strategy converts some of those scrapped or below-quality tons into additional product output ΔP, which increases the value of Y to Y_f, we will not only increase process A's throughput but will also "recover" $P(Y_f-Y)Et$ units of energy since $Et\left(1/Y - 1/Y_f\right)$ fewer units of energy per ton would be required to get acceptable-quality product out of process A. It is important to note that under the second scenario, process A is still consuming $P \times Et$ units of energy, which illustrates the difference between the two improvement approaches.

The VCA model's sensitivity parameters fall under the second approach as follows:

- $\left(\Delta C/\Delta P\right)$ represents the amount of incurred production cost recovered per unit of product output reclaimed.
- $\left(\Delta P/\Delta E\right)$ represents the amount of product recovered per unit of energy reclaimed.
- $\left(\Delta E/\Delta \eta\right)$ represents the amount of energy recovered per percentage unit of lost material reclaimed.
- $\left(\Delta \eta/\Delta E\right)$ represents the percentage of lost material recovered per unit of energy reclaimed.

Given that sensitivity parameters represent potential savings through the enhanced utilization of the production process' existing resources, the basic enterprise optimization approach uses a two-step procedure. The first step estimates benefit coefficients to quantify the potential benefit that the improvement of each process offers. The second step uses the benefit coefficients, along with operational parameters and any other relevant information for each feasible operational strategy, to select a group of new operational strategies that optimize the enterprise's performance as defined by the objective function while meeting a set of constraints.

The procedure explained above provides three major advantages. It forces us to understand and evaluate the production process' needs and potential first. It then allows us to evaluate a diversified group of operational strategies on a leveled playing field based on a specific production process. Finally, it takes into account the global system to assure improved performance for the entire enterprise rather than focusing on local optima.

3.2 Type 1 EMP Model to Increase Throughput and Maximize Profit

3.2.1 Step 1: Estimating EBCs

An EBC (Economic Benefit Coefficient) measures the financial benefit, at an enterprise level, per ton of product output recovered at a given process. If P_I, P_O, and Y represent a process' material input, material output, and yield $\left(P_O/P_I\right)$, respectively; and a new operational strategy generates an additional ΔP material output, which will increase Y to $\hat{Y}$ $\left(\frac{P_O + \Delta P}{P_I}\right)$; the additional material output is not only bound to generate additional revenue, but it will also make possible to allocate the process' total cost among a larger amount of acceptable-quality product, which reduces the resulting cost per ton. Consequently, under this philosophy, an EBC has two components: the recovered cost from the current throughput and the additional profit from the reclaimed material.

The recovered cost at j^{th} process can be expressed as either

$$\frac{\left(\frac{\Delta C}{\Delta P}\right)_j \Delta P_j}{\hat{Y}_j} \tag{20}$$

or

$$\sum_{i=1}^{n} (C_l - \hat{C}_l)\, Ti_{(l)} \tag{21}$$

$$\text{where} \quad \hat{Y}_j = Y_j + \frac{\Delta P_j}{\sum_{k=0}^{m} P_{k,j}} \tag{22}$$

represents the improved yield, and $\hat{C}_l$ represents the reduced total accumulated cost per ton of a given $Ti_{(l)}$.

Breaking up the summation in (21), we can calculate $\hat{C}_l$ from Eq. (20) and Eq. (21) as

$$\hat{C}_l = C_l - \frac{\left(\frac{\Delta C}{\Delta P}\right)_j \Delta P_j \prod_{h,k} r_{h,k,i}}{\hat{Y}_j Ti_{(l)}} \tag{23}$$

where $\prod_{h,k} r_{h,k,i}$ represents the compounded ratio from j^{th} process to i^{th} final product.

On the other hand, the additional profit from the reclaimed material at j^{th} process can be expressed as

$$\sum_{i=1}^{n} (R_i - \hat{C}_l)\, \Delta Ti_{(l)} \tag{24}$$

$$\text{where} \quad \Delta Ti_{(l)} = \Delta P_j \prod_{h,k} r_{h,k,i} \prod_{k} Y_k \tag{25}$$

and $\prod_{k} Y_k$ represents the compounded yield from j^{th} process to i^{th} final product.

As a result, the economic benefit (EB) for j^{th} process can be calculated by adding Eq. (20) and Eq. (24),

$$EB_j = \frac{\left(\frac{\Delta C}{\Delta P}\right)_j \Delta P_j}{\hat{Y}_j} + \Delta P_j \sum_{i=1}^{n} \left((R_i - \hat{C}_l) \prod_{h,k} r_{h,k,i} \prod_{k} Y_k \right) \tag{26}$$

However, if we assume that ΔP_j does not absorb any preceding cost and, as a result, comes out of j^{th} process with a zero accumulated cost while the already existing throughput $\sum_{k=1}^{m} P_{j,k}$ continues to absorb the total accumulated cost; EB_j can also be calculated as

$$EB_j = \sum_{i=1}^{n} (R_i - C'_l)\, \Delta Ti_{(l)} = \Delta P_j \sum_{i=1}^{n} \left((R_i - C'_l) \prod_{h,k} r_{h,k,i} \prod_{k} Y_k \right) \tag{27}$$

where C'_l represents the reduced total accumulated cost per ton of ΔP_j that becomes a given $Ti_{(l)}$ when C_j is made equal to zero.

Therefore, EBs can be calculated by either replacing Eq. (22) and Eq. (23) in Eq. (26),

$$EB_j = \frac{\left(\frac{\Delta C}{\Delta P}\right)_j \left(\sum_{k=0}^{m} P_{k,j}\right) \Delta P_j}{Y_j \sum_{k=0}^{m} P_{k,j} + \Delta P_j} + \Delta P_j \sum_{i=1}^{n} \left(\left(R_i - C_l + \frac{\left(\frac{\Delta C}{\Delta P}\right)_j \prod_{h,k} r_{h,k,i} \left(\sum_{k=0}^{m} P_{k,j}\right) \Delta P_j}{\left(Y_j \sum_{k=0}^{m} P_{k,j} + \Delta P_j\right) Ti_{(l)}} \right) \prod_{h,k} r_{h,k,i} \prod_{k} Y_k \right) \tag{28}$$

or by utilizing Eq. (27), which not only presents obvious computational advantages but also allows us to easily calculate EBCs as

$$EBC_j = \frac{EB_j}{\Delta P_j} = \sum_{i=1}^{n} \left((R_i - C'_l) \prod_{h,k} r_{h,k,i} \prod_{k} Y_k \right) \tag{29}$$

It is important to mention that EBC values may change as new operational strategies are selected. In fact, the introduction of a new operational strategy makes all new "upstream" operational strategies more attractive, although it does not affect the economical desirability of all new "downstream" operational strategies. The former can be proven by realizing that in Eq. (29) the values of R_i and $\prod_{h,k} r_{h,k,i}$ remain constant; but the value of $\prod_{k} Y_k$ increases, which makes C'_l smaller, resulting in a larger EBC_j. The latter is true since all terms in Eq. (29) are unchanged by the introduction of new "upstream" operational strategy.

3.2.2 Step 2: Operational Strategy Selection

The decision variable for the Step 2 EMP model is the binary variable $OS(j,a)$ that indicates whether the a^{th} new operational strategy has been implemented in the j^{th} process as follows,

$$OS(j,a) = \begin{cases} 1 & \text{if the } a^{th} \text{ new operational strategy for the } j^{th} \text{ process has been chosen} \\ 0 & \text{otherwise} \end{cases} \qquad (30)$$

It is important to note that each variable $OS(j,a)$ represents a 1-1 function between the strategy and the process since a given strategy is expected to be applicable to one and only one specific process. In other words, if q operational strategies are to be considered, the index a will take on values from 1 to q and no two $OS(j,a)$ variables will have the same a value.

Nevertheless, if a strategy can be implemented in more than one process, a separate $OS(j,a)$ variable will be assigned for each strategy-process possibility. For instance, if the first strategy to be considered for the process in figure 6 is applicable to processes 2, 4 and 5; three variables such as $OS(2,1)$, $OS(4,2)$ and $OS(5,3)$ will be included in the model. Obviously, under this scenario, the maximum value of a will be greater than q.

At least three groups of constraints have to be considered at this step: upper-bound, capacity, and budget constraints. However, due to the fact that additional material flow will be the result of adopting new operational strategies, the following secondary decision variable will be used to simplify the construction of the constraints,

$$\Delta P_j = \sum_{a=1}^{q} \left(\Delta P_{j(a)} \, OS(j,a) \right) \qquad \text{for } j=1,2,\ldots,m \qquad (31)$$

where $\Delta P_{j(a)}$ represents the amount of material recovered by the a^{th} new operational strategy at the j^{th} process, and ΔP_j indicates the total amount of material recovered at the j^{th} process.

Upper-bound constraints establish upper limits on each one of the secondary decision variables as follows,

$$\hat{Y}_j = Y_j + \frac{\Delta P_j}{\sum_{k=0}^{m} P_{k,j}} \leq 1 \qquad \text{for } j=1,2,\ldots,m \qquad (32)$$

If K_j for $j=1,2,\ldots,m$ represents the maximum processing capacity at j^{th} process, and $\widetilde{P}_j$ for $j=1,2,\ldots,m$ represents the additional amount of product that j^{th} process will receive from its predecessors, capacity constraints can be expressed as follows:

$$\sum_{k=1}^{m} P_{k,j} + \widetilde{P}_j \leq K_j \qquad (33)$$

$$\text{where} \quad \widetilde{P}_j = \sum_{l=1}^{m} \left[r_{l,j,0} \left(\Delta P_l + \hat{Y}_l \, \widetilde{P}_l \right) \right] \qquad \text{for } j=1,2,\ldots,m \qquad (34)$$

If B represents the firm's available investment budget and I_a is the expected capital investment for a^{th} new operational strategy, a budget constraint takes the form

$$\sum_{a=1}^{q} \left(I_a \, OS(j,a) \right) \leq B \qquad (35)$$

It is important to note that if the capital investment of a given strategy is spread over a number of periods (months or years), I_a is equivalent to the NPV of those cash outflows. Finally, additional constraints may include, for instance, a limitation on the maximum total energy consumption *Em* as follows,

$$\sum_{j=1}^{m}\left(\left(\sum_{k=1}^{m}P_{k,j}+\widetilde{P}_j\right)Et_j\right)\leq Em \tag{36}$$

The main objective function will be to maximize the enterprise's financial performance. In order to achieve this, the annual worth method will be used to analyze the economic impact of each new operational strategy on the enterprise's financial performance and to compare all new operational strategies under the assumption that they are independent in nature, which is discussed later. The annual worth (AW) for the a^{th} new operational strategy is defined as

$$AW_a = CF_a - I_a(A/P, i\%, N) \tag{37}$$

where CF_a is the annual cash flow, which will be estimated as

$$CF_a = \text{Annual Benefit} - \text{Annual Expenses} = EBC_j\ \Delta P_{j(a)} - AE_a \tag{38}$$

$i\%$ is the enterprise's minimum attractive rate of return (MARR); N is the time horizon for economic analysis, expressed in years;

$$(A/P, i\%, N) = \frac{i(1+i)^N}{(1+i)^N - 1} \tag{39}$$

is the capital recovery factor; and AE_a is the expected annual expense for a^{th} new operational strategy.

It should be noted that Eq. (37) assumes that:

(a) All new operational strategies either have a common useful life N or the positive cash flows or shorter-lived operational strategies are reinvested at the MARR over a period of time corresponding to the life of the longest-lived operational strategy to enable the fair comparison of alternatives;

(b) Salvage (residual) value for all operational strategies at the end of the study period is zero;

(c) Annual benefit and expenses will experience a common price inflation rate and are, consequently, expressed in real dollars (dollars expressed in terms of the same purchasing power relative to the year in which the study is conducted); and

(d) $i\%$ represents the real MARR (an effective rate per interest period that does not include a market adjustment for the anticipated general price inflation rate in the economy).

The main objective function is then expressed as

$$\text{Max} \quad \sum_{a=1}^{q} \left(AW_a \, OS(j,a) \right) \tag{40}$$

Since all operational strategies are considered to be independent and a budget constraint has been introduced, selecting strategies with the highest AWs will not guarantee an enterprise optimum even when equivalent worth methods such as AW are used (Park and Sharp-Bette, 1990). Therefore, it is necessary to define mutually exclusive investment alternatives in terms of combination of operational strategies, and these investment alternatives have to be analyzed through the incremental or total investment analysis in figure 4.

Although the first step is not required, it may prove to be very significant as follows: if q operational strategies are available, there will be 2^q possible investment alternatives; so each operational strategy eliminated in step 1 diminishes the number of investment alternatives to be evaluated in half. Given the fact that, as discussed previously, EBC values may increase due to the selection of other operational strategies; the first step can be carried out by calculating the AW while setting $\prod_k Y_k$ equal to 1 in Eq. (29), which yields the maximum EBC values.

Furthermore, the net cash flow of each investment combination is determined by simply adding, period by period, the cash flows of each operational strategy included in the combination. Finally, an investment alternative will be deemed feasible if the implementation of its operational strategies violates neither upper-bound nor capacity constraints. In other words, no fractional acceptance of an operational strategy will be allowed since it changes the nature of the operational strategy's recurring cash flows –specifically, the estimation of CF_a in Eq. (38).

3.2.3 Model Statement

Maximize $\sum_{a=1}^{q}\left(AW_a\, OS(j,a)\right)$

Subject to

$$OS(j,a) = \begin{cases} 1 & \text{if the } a^{\text{th}} \text{ new operational strategy for the } j^{\text{th}} \text{ process has been chosen } (a=1,2,\ldots,q) \\ 0 & \text{otherwise} \end{cases}$$

$\hat{Y}_j \le 1$ for $j=1,2,\ldots,m$ (Upper-bound)

$\sum_{k=1}^{m} P_{k,j} + \widetilde{P}_j \le K_j$ for $j=1,2,\ldots,m$ (Capacity)

$\sum_{a=1}^{q}\left(I_a\, OS(j,a)\right) \le B$ (Budget)

3.2.4 Numerical Example

The sensitivity parameters, which are calculated using the equations included in Appendix 1, are presented in table 4, which also includes the process parameters for each one of the processes in figure 6's hypothetical example. Moreover, the indicators and process impacts, as defined by the VCA model in the appendix, for 23 different hypothetical operational strategies are shown in table 5. It is important to note that Y_j, K_j, Et_j, Ft_j, Vt_j and ΔP, whose values are presented in bold font style, are input variables required by the EMP model to calculate the other variable values. Table 6, in turn, presents the initial EBC values.

The upper-bound constraints for the hypothetical production process are

$$\hat{Y}_1, \hat{Y}_2, \ldots, \hat{Y}_{12} \le 1$$

Likewise, these are the capacity constraints using data from tables 2 and 4 (note that $\widetilde{P}_1 = 0$):

$$\widetilde{P}_2 \le 200$$
$$\widetilde{P}_3 \le 298$$

Table 4. Numerical Example for Sensitivity Parameters (Type 1 EMP Model)

PROCESS	PROCESS PARAMETERS							SENSITIVITY PARAMETERS			
	Y_j	$\sum_k P_{k,j}$	K_j	Et_j	Ft_j	Vt_j	$PDCt_j$	$\left(\frac{\Delta P}{\Delta E}\right)_j$	$\left(\frac{\Delta C}{\Delta P}\right)_j$	$\left(\frac{\Delta E}{\Delta \eta}\right)_j$	$\left(\frac{\Delta \eta}{\Delta C}\right)_j$
1	**0.98**	10,000	**10,500**	**20**	**7**	**5**	25.0	0.0500	37.0	4,000	0.000135
2	**0.99**	9,800	**10,000**	**16**	**10**	**8**	37.8	0.0625	55.8	1,568	0.000183
3	**0.98**	9,702	**10,000**	**22**	**10**	**4**	56.3	0.0455	70.3	4,269	0.000073
4	**0.97**	9,508	**9,750**	**18**	**4**	**8**	71.8	0.0556	83.8	5,134	0.000042
5	**0.96**	8,300	**8,750**	**22**	**8**	**6**	86.3	0.0455	100.3	7,304	0.000030
6	**0.93**	2,391	**2,450**	**15**	**4**	**7**	104.5	0.0667	115.5	2,511	0.000052
7	**0.92**	2,223	**2,250**	**21**	**5**	**3**	124.2	0.0476	132.2	3,735	0.000043
8	**0.99**	2,045	**2,250**	**19**	**5**	**2**	143.7	0.0526	150.7	389	0.000324
9	**0.97**	5,578	**6,000**	**21**	**3**	**5**	104.5	0.0476	112.5	3,514	0.000053
10	**0.95**	2,705	**3,000**	**17**	**6**	**3**	116.0	0.0588	125.0	2,299	0.000059
11	**0.94**	2,570	**2,750**	**21**	**9**	**2**	131.6	0.0476	142.6	3,238	0.000045
12	**0.91**	1,623	**1,750**	**16**	**4**	**4**	116.0	0.0625	124.0	2,337	0.000055

Table 5. Numerical Example for Operational Strategies (Type 1 EMP Model)

PROCESS	OPERATIONAL STRATEGY	INDICATORS		PROCESS IMPACTS			I_a	AE_a
		ΔP	ΔPDC	ΔE	$\Delta \eta$	ΔPCC		
1	OS[1,1]	**54**	1,350	1,080	0.270	648	**17,500**	**1,050**
1	OS[1,2]	**51**	1,275	1,020	0.255	612	**18,500**	**900**
1	OS[1,3]	**84**	2,100	1,680	0.420	1,008	**14,000**	**975**
2	OS[2,4]	**55**	2,077	880	0.561	990	**16,000**	**935**
3	OS[3,5]	**86**	4,843	1,892	0.443	1,204	**25,000**	**1,555**
3	OS[3,6]	**74**	4,168	1,628	0.381	1,036	**38,500**	**1,960**
4	OS[4,7]	**45**	3,229	810	0.158	540	**12,500**	**955**
4	OS[4,8]	**100**	7,175	1,800	0.351	1,200	**26,000**	**1,950**
4	OS[4,9]	**38**	2,727	684	0.133	456	**21,500**	**1,750**
4	OS[4,10]	**23**	1,650	414	0.081	276	**13,500**	**925**
5	OS[5,11]	**42**	3,626	924	0.127	588	**10,500**	**675**
6	OS[6,12]	**33**	3,449	495	0.197	363	**11,000**	**450**
6	OS[6,13]	**21**	2,195	315	0.125	231	**13,000**	**1,350**
6	OS[6,14]	**12**	1,254	180	0.072	132	**11,500**	**1,225**
7	OS[7,15]	**20**	2,484	420	0.112	160	**17,500**	**675**
8	OS[8,16]	**15**	2,156	285	0.733	105	**10,500**	**1,300**
9	OS[9,17]	**35**	3,658	735	0.209	280	**12,000**	**480**
9	OS[9,18]	**63**	6,585	1,323	0.376	504	**21,500**	**1,280**
9	OS[9,19]	**14**	1,463	294	0.084	112	**14,500**	**275**
10	OS[10,20]	**20**	2,320	340	0.148	180	**19,000**	**625**
11	OS[11,21]	**36**	4,737	756	0.233	396	**12,000**	**520**
11	OS[11,22]	**22**	2,895	462	0.143	242	**21,000**	**950**
12	OS[12,23]	**20**	2,320	320	0.137	160	**15,750**	**595**

Table 6. Initial and Adjusted EBCs (Type 1 EMP Model Under Increase-Throughput, Maximize-Profit Philosophy).

	Process Parameters			C'_l Reduced accumulated cost when process' $C_j = 0$					$\prod_{h,k} r_{h,k,i}$ Compounded Ratio to Product					$\prod_k Y_k$ Compounded Yield to Product					EBC Contribution from Product					Initial EBC_j
PROCESS	Y_j	Ft_j	Vt_j	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5	
1	0.98	7	5	46.2	72.9	102.9	103.4	88.9	0.100	0.126	0.270	0.315	0.189	0.941	0.876	0.765	0.783	0.797	5.06	6.85	12.83	17.64	8.45	50.838
2	0.99	10	8	27.1	52.4	79.4	80.4	66.4	0.100	0.126	0.270	0.315	0.189	0.951	0.885	0.773	0.790	0.806	6.93	9.22	17.87	23.55	11.97	69.534
3	0.98	10	4	12.4	36.6	61.3	62.7	49.0	0.100	0.126	0.270	0.315	0.189	0.970	0.903	0.789	0.807	0.822	8.50	11.20	22.09	28.53	14.92	85.238
4	0.97	4	8	0.0	23.3	46.1	47.9	34.4	0.100	0.126	0.270	0.315	0.189	1.000	0.931	0.813	0.832	0.847	10.00	13.11	26.11	33.31	17.72	100.244
5	0.96	8	6		8.2	28.8	31.0	17.9		0.140	0.300	0.350	0.210		0.970	0.847	0.866	0.883	0.00	17.21	34.60	43.65	23.57	119.033
6	0.93	4	7			15.9					1.000					0.911			0.00	0.00	135.85	0.00	0.00	135.846
7	0.92	5	3			7.1					1.000					0.990			0.00	0.00	156.35	0.00	0.00	156.351
8	0.99	5	2			0.0					1.000					1.000			0.00	0.00	165.00	0.00	0.00	165.000
9	0.97	3	5		0.0		21.8	8.8		0.200		0.500	0.300		1.000		0.893	0.910	0.00	27.00	0.00	68.41	37.19	132.598
10	0.95	6	3				11.7					1.000					0.940		0.00	0.00	0.00	153.50	0.00	153.502
11	0.94	9	2				0.0					1.000					1.000		0.00	0.00	0.00	175.00	0.00	175.000
12	0.91	4	4					0.0					1.000					1.000	0.00	0.00	0.00	0.00	145.00	145.000

R_i	100	135	165	175	145

	EBC Contribution from Product with $\prod_k Y_k = 1$					Adjusted EBC_j
PROCESS	1	2	3	4	5	
1	5.38	7.82	16.77	22.54	10.60	63.105
2	7.29	10.41	23.12	29.79	14.86	85.469
3	8.76	12.40	28.01	35.37	18.15	102.687
4	10.00	14.08	32.11	40.05	20.91	117.150
5	0.00	17.75	40.85	50.39	26.70	135.688
6	0.00	0.00	149.15	0.00	0.00	149.150
7	0.00	0.00	157.93	0.00	0.00	157.930
8	0.00	0.00	165.00	0.00	0.00	165.000
9	0.00	27.00	0.00	76.61	40.86	144.473
10	0.00	0.00	0.00	163.30	0.00	163.300
11	0.00	0.00	0.00	175.00	0.00	175.000
12	0.00	0.00	0.00	0.00	145.00	145.000

$$\begin{aligned}
\widetilde{P}_4 &\leq 242 \\
\widetilde{P}_5 &\leq 450 \\
\widetilde{P}_6 &\leq 59 \\
\widetilde{P}_7 &\leq 27 \\
\widetilde{P}_8 &\leq 205 \\
\widetilde{P}_9 &\leq 422 \\
\widetilde{P}_{10} &\leq 295 \\
\widetilde{P}_{11} &\leq 180 \\
\widetilde{P}_{12} &\leq 127
\end{aligned} \tag{41}$$

where, for instance, $\widetilde{P}_4 = \Delta P_3 + \hat{Y}_3 \Delta P_2 + \hat{Y}_3 \hat{Y}_2 \Delta P_1$ with $\hat{Y}_2 = 0.99 + \dfrac{\Delta P_2}{9{,}800}$ and $\hat{Y}_3 = 0.98 + \dfrac{\Delta P_3}{9{,}702}$, which makes Eq. (41) equivalent to

$$0.9702\ \Delta P_1 + 0.98\ \Delta P_2 + \Delta P_3 + \frac{\Delta P_1 \Delta P_2}{10{,}000} + \frac{\Delta P_1 \Delta P_3}{9{,}800} + \frac{\Delta P_2 \Delta P_3}{9{,}702} + \frac{\Delta P_1 \Delta P_2 \Delta P_3}{95{,}079{,}600} \leq 242$$

Two more constraints are added. The first one keeps the selection of new operational strategies within budget limitations,

$$\sum_{a=1}^{23} \left[I_a\ OS(j,a) \right] \leq 70{,}000 \tag{42}$$

-the last two columns in table 5 exhibit the initial capital investment and annual expense values for each hypothetical new operational strategy. The second constraint sets limits on the decision variables following Eq. (30),

$$OS(j,a) \in \{0,1\} \qquad \text{for } a=1,2,\ldots,23 \tag{43}$$

Assuming that $i\%$ and N are equal to 12% and 15 years, respectively; we can calculate AW for each operational strategy using the adjusted EBC values presented at the bottom of table 6 as indicated by the first step of the figure 4. The results are shown in table 7, which indicates that

Table 7. Initial AW Calculations Assuming 100% Downstream Yield (Type 1 EMP Model Under Increase-Throughput, Maximize-Profit Philosophy).

Operational Strategy	AW_a
1	-211.77
2	-397.91
3	2,270.26
4	1,416.59
5	3,605.48
6	-13.90
7	2,481.44
8	5,947.56
9	-455.02
10	-212.68
11	3,482.22
12	2,865.88
13	-126.57
14	-1,123.68
15	-85.82
16	-366.65
17	2,814.66
18	4,665.08
19	-381.33
20	-148.66
21	4,018.11
22	-183.31
23	-7.48

only 10 of the 23 operational strategies should be used to form 1,024 (2^{10}) investment alternatives. However, since 670 alternatives exceed the budget constraint in Eq. (42), only 354 alternatives have to be checked for feasibility. Although no alternatives violate the upper-bound constraints, 279 alternatives do exceed some capacity constraints –primarily those of processes 6 and 7-, which leaves 75 feasible investment alternatives. Table 8 shows the AW for each feasible alternative (Total investment approach, steps 13 to 15 in figure 4) sorted from highest to lowest.

The set of new operational strategies that maximizes the financial performance of the hypothetical production process in figure 6 is OS(4,7), OS(5,11), OS(9,17), OS(9,18) and OS(11,21), which requires a \$68,500 investment and generates an additional annual profit of \$25,230, resulting in a positive AW of \$15,172.54. If the operational strategies with the highest AW values had been selected, without violating any constraints, the resulting set would have been comprised of *OS*(4,8), *OS*(9,18) and *OS*(11,21) –the fourth investment alternative in table 8-, which is sub optimal with an AW of \$12,424.14.

3.3 Type 1 EMP Model to Maintain Throughput and Maximize Profit

3.3.1 Step 1: Estimating EBCs

EBCs under this philosophy only exhibit one component: the reduced material input cost. If P_I, P_O, and Y represent a process' material input, material output, and yield $\left(P_O/_{P_I}\right)$, respectively; if a new operational strategy has the potential of generating an additional ΔP material output, which will increase Y to $\hat{Y}$ $\left(\dfrac{P_O+\Delta P}{P_I}\right)$; and if P_O is to be maintained; then an extra ΔP material output is equivalent to an upstream $\Delta P/_{\hat{Y}}$ material input reduction. This reduced amount of material not only saves the total accumulated cost up through the previous process, but also saves the fixed and variable costs to be incurred at this process; both of which are measured by the cost-to-product sensitivity parameter $\dfrac{\Delta C}{\Delta P}$. Consequently,

Table 8. AW Calculations for Feasible Investment Alternatives (Type 1 EMP Model Under Increase-Throughput, Maximize-Profit Philosophy).

	OPERATIONAL STRATEGIES [1]										AW
	3	4	5	7	8	11	12	17	18	21	
1	0	0	0	1	0	1	0	1	1	1	15,172.54
2	0	0	0	0	0	1	0	1	1	1	13,332.22
3	0	0	0	1	0	1	0	0	1	1	12,685.12
4	0	0	0	0	1	0	0	0	1	1	12,424.14
5	0	0	0	1	0	0	0	1	1	1	12,262.59
6	1	0	0	0	0	0	0	1	1	1	11,853.34
7	0	1	0	0	0	1	0	0	1	1	11,520.16
8	0	1	0	0	0	0	0	1	1	1	11,097.31
9	0	0	0	1	0	1	0	1	0	1	11,096.51
10	0	0	0	1	0	1	0	1	1	0	10,975.05
11	0	0	0	0	0	1	0	0	1	1	10,867.77
12	0	0	0	0	1	0	0	1	0	1	10,833.03
13	0	0	0	0	1	0	0	1	1	0	10,712.92
14	0	1	0	1	0	0	0	0	1	1	10,449.56
15	0	0	0	0	0	0	0	1	1	1	10,447.16
16	0	0	1	0	0	0	0	0	1	1	10,245.43
17	0	1	0	0	0	1	0	1	0	1	9,928.57
18	0	1	0	0	0	1	0	1	1	0	9,808.71
19	0	0	0	1	0	0	0	0	1	1	9,799.97
20	1	0	0	0	0	0	0	0	1	1	9,373.44
21	0	0	0	0	0	1	0	1	0	1	9,297.52
22	0	0	0	0	0	1	0	1	1	0	9,166.19
23	0	1	0	1	0	0	0	1	0	1	8,859.45
24	0	1	0	1	0	0	0	1	1	0	8,738.79
25	0	0	1	0	0	0	0	1	0	1	8,661.05
26	0	1	0	0	0	0	0	0	1	1	8,631.00
27	0	0	0	1	0	1	0	0	0	1	8,609.09
28	0	0	1	0	0	0	0	1	1	0	8,537.32
29	0	0	0	1	0	1	0	0	1	0	8,528.80
30	0	0	0	0	1	0	0	0	0	1	8,342.49
31	0	0	0	0	1	0	0	0	1	0	8,263.57
32	0	0	0	1	0	0	0	1	0	1	8,231.19
33	0	0	0	1	0	0	0	1	1	0	8,099.07
34	0	0	0	0	0	0	0	0	1	1	8,007.39
35	1	0	0	0	0	0	0	1	0	1	7,790.83
36	1	0	0	0	0	0	0	1	1	0	7,666.15
37	0	1	0	0	0	1	0	0	0	1	7,437.44
38	0	1	0	0	0	1	0	0	1	0	7,358.78

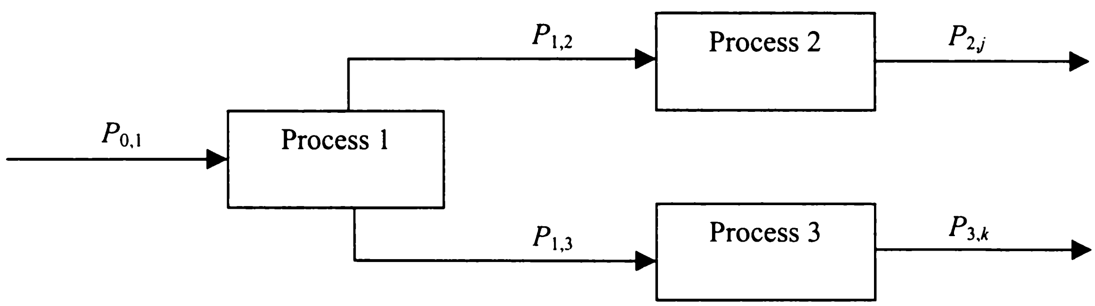
	OPERATIONAL STRATEGIES [1]										AW
	3	4	5	7	8	11	12	17	18	21	
39	0	1	0	0	0	0	0	1	0	1	7,059.26
40	0	0	0	1	0	1	0	1	0	0	6,973.12
41	0	1	0	0	0	0	0	1	1	0	6,928.73
42	0	0	0	0	0	1	0	0	0	1	6,833.07
43	0	0	0	0	0	1	0	0	1	0	6,742.71
44	0	0	0	0	1	0	0	1	0	0	6,705.42
45	0	0	0	0	0	0	0	1	0	1	6,456.88
46	0	1	0	1	0	0	0	0	0	1	6,370.16
47	0	0	0	0	0	0	0	1	1	0	6,314.94
48	0	1	0	1	0	0	0	0	1	0	6,290.69
49	0	0	1	0	0	0	0	0	0	1	6,178.93
50	0	0	1	0	0	0	0	0	1	0	6,096.32
51	0	1	0	0	0	1	0	1	0	0	5,800.15
52	0	0	0	1	0	0	0	0	0	1	5,768.57
53	0	0	0	1	0	0	0	0	1	0	5,677.41
54	1	0	0	0	0	0	0	0	0	1	5,310.93
55	1	0	0	0	0	0	0	0	1	0	5,227.34
56	0	0	0	0	0	1	0	1	0	0	5,205.24
57	0	1	0	1	0	0	0	1	0	0	4,733.52
58	0	1	0	0	0	0	0	0	0	1	4,592.95
59	0	0	1	0	0	0	0	1	0	0	4,544.84
60	0	0	0	1	0	1	0	0	0	0	4,526.88
61	0	1	0	0	0	0	0	0	1	0	4,503.40
62	0	0	0	0	1	0	0	0	0	0	4,256.07
63	0	0	0	1	0	0	0	1	0	0	4,141.39
64	0	0	0	0	0	0	0	0	0	1	4,017.11
65	0	0	0	0	0	0	0	0	1	0	3,915.92
66	1	0	0	0	0	0	0	1	0	0	3,677.62
67	0	1	0	0	0	1	0	0	0	0	3,350.22
68	0	1	0	0	0	0	0	1	0	0	2,964.46
69	0	0	0	0	0	1	0	0	0	0	2,781.76
70	0	0	0	0	0	0	0	1	0	0	2,398.02
71	0	1	0	1	0	0	0	0	0	0	2,285.42
72	0	0	1	0	0	0	0	0	0	0	2,103.84
73	0	0	0	1	0	0	0	0	0	0	1,719.72
74	1	0	0	0	0	0	0	0	0	0	1,238.82
75	0	1	0	0	0	0	0	0	0	0	539.12

[1] "1" indicates that an Operational Strategy is included in the Investment Alternative.

$$EB_j = \frac{\left(\frac{\Delta C}{\Delta P}\right)_j \Delta P_j}{\hat{Y}_j} = \frac{\left(\frac{\Delta C}{\Delta P}\right)_j \Delta P_j}{Y_j + \frac{\Delta P_j}{\sum_{k=0}^{m} P_{k,j}}} \tag{44}$$

after replacing Eq. (22). However, when EB_j is divided by ΔP_j to calculate EBC_j, the result is still dependent upon ΔP_j, which means that it is not possible to derive a value for EBC_j that is the same for all new operational strategies that could be implemented at the j^{th} process as we did in the previous philosophy. In other words, under this philosophy, an EBC measures the financial benefit, at an enterprise level, per ton of product output that would have been recovered at a given process *by a given operational strategy*. This is due to the fact that the marginal EBC value for each additional ton of material recovered monotonically declines. In any case, if two operational strategies have the same $\Delta P_{j(a)}$ values, their EBC value will still be identical.

Furthermore, as new operational strategies are selected, the EM model will have to be rebalanced in order to maintain final product throughputs constant. For example, if an operational strategy is selected for process 2 in the illustration below, less $P_{1,2}$ and $P_{0,1}$ material flow is needed to maintain the value of $P_{2,j}$. However, if $r_{1,2,0}$ and $r_{1,3,0}$ are not adjusted down and up, respectively, $P_{2,j}$ will become larger while $P_{3,k}$ will become smaller.



The Type 1 EM model's rebalancing process will be performed under the following guidelines:

1. No rebalancing is needed downstream of an improved process.

2. If a preceding j^{th} process divides its material output into two or more processes, or into a process and a final product, its $r_{j,k,i}$ values will have to be recalculated as illustrated below.

Once again, if a new operational strategy is implemented in process 2 above, the value of $P_{2,j}$ remains constant while the value of $P_{1,2}$ is reduced to $\hat{P}_{1,2}$. To keep $P_{2,j}$ and $P_{3,k}$ constant, process 1's ratios are recalculated as

$$r_{1,2,0} = \frac{\hat{P}_{1,2}}{\hat{P}_{1,2} + P_{1,3}} \qquad \text{and} \qquad r_{1,3,0} = \frac{P_{1,3}}{\hat{P}_{1,2} + P_{1,3}}$$

Finally, as before, EB values may change as new operational strategies are selected. In fact, the introduction of a strategy makes all remaining "downstream" strategies less economically attractive since $\Delta C / \Delta P$ in Eq. (44) reduces when "upstream" processes become more efficient. Likewise, since the adoption of a strategy results in a reduction of "upstream" flow, "upstream" EBs also decrease since the potential material savings diminishes as the material input P_l lowers. In conclusion, under this philosophy, all EB values decline as strategies are introduced.

3.3.2 Step 2: Operational Strategy Selection

Although capacity constraints are not needed under this philosophy, upper-bound and budget constraints are still required. In fact, upper-bound constraints are needed because, although throughput is not going to increase under this philosophy, the yield for a given process cannot exceed 100%. Moreover, the main objective function is still to maximize the enterprise's financial performance using the annual worth method as defined in Eq. (37). However, the annual cash flow for $OS(j,a)$ will now be estimated as

$$CF_a = \text{Annual Benefit} - \text{Annual Expenses} = EB_j - AE_a \qquad (45)$$

3.3.3 Model Statement

Maximize $\quad \sum_{a=1}^{q} \left(AW_a \; OS(j,a) \right)$

Subject to

$$OS(j,a) = \begin{cases} 1 & \text{if the } a^{\text{th}} \text{ new operational strategy for the } j^{\text{th}} \text{ process has been chosen} \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{Y}_j \leq 1 \qquad \text{for } j=1,2,\ldots,m \quad \text{(Upper-bound)}$$

$$\sum_{a=1}^{q} \left[I_a \, OS(j,a) \right] \leq B \qquad \text{(Budget)}$$

In the upper-bound constraint above, $\hat{Y}_j$ is calculated as specified in Eq. (22) with ΔP_j representing the amount of material that *would have been recovered* at the j^{th} process had the decision been made to increase throughput.

3.3.4 Numerical Example

The constraints below, along with the process parameters, indicators, and economic data in tables 4 and 5, will constitute the foundation for the figure 6's production process model statement:

$$OS(j,a) \in \{0,1\} \qquad \text{for } a=1,2,\ldots,23$$

$$\hat{Y}_1, \hat{Y}_2, \ldots, \hat{Y}_{12} \leq 1 \qquad \text{(Yield)}$$

$$\sum_{a=1}^{23} \left[I_a \, OS(j,a) \right] \leq 70{,}000 \qquad \text{(Budget)}$$

We first calculate the AW for each operational strategy, assuming that *i%* and *N* are equal to 12% and 15 years, which allows us to eliminate 14 of the 23 strategies as shown in table 9. At this point, 512 (2^9) investment alternatives are formed. However, there are only 229 feasible alternatives since 283 exceed the budget constraint and none violate any of the yield constraints.

Table 10 shows the AW for the 17 feasible alternatives whose AW is greater than $9,000, as calculated using the total investment approach -steps 13 to 15 in figure 4. The set of new operational strategies that maximizes financial performance is then *OS*(5,11), *OS*(6,12), *OS*(9,17),

Table 9. Initial AW Calculations (Type 1 EMP Model Under Maintain-Throughput, Maximize-Profit Philosophy).

Operational Strategy	AW_a
1	-1,591.82
2	-1,700.71
3	113.94
4	-204.14
5	889.87
6	-2,343.99
7	1,076.30
8	2,774.32
9	-1,639.13
10	-926.16
11	2,150.36
12	1,974.25
13	-674.50
14	-1,430.84
15	-397.91
16	-574.85
17	1,792.18
18	2,787.46
19	-784.08
20	-803.30
21	3,098.59
22	-726.35
23	-218.51

Table 10. AW Calculations for Top 17 Feasible Investment Alternatives (Type 1 EMP Model Under Maintain-Throughput, Maximize-Profit Philosophy).

	OPERATIONAL STRATEGIES [1]									AW
	3	5	7	8	11	12	17	18	21	
1	0	0	0	0	1	1	1	1	1	11,533.35
2	0	0	1	0	1	1	0	1	1	10,878.50
3	0	0	1	0	1	0	1	1	1	10,575.51
4	0	0	0	1	1	0	0	1	1	10,559.36
5	0	0	1	0	0	1	1	1	1	10,478.56
6	0	0	1	0	1	1	1	0	1	9,934.22
7	1	0	0	0	1	1	0	1	1	9,930.88
8	0	0	0	0	1	1	0	1	1	9,881.12
9	0	0	0	1	1	1	0	0	1	9,842.62
10	0	0	0	1	1	0	1	0	1	9,629.86
11	1	0	0	0	1	0	1	1	1	9,627.88
12	0	0	0	0	1	0	1	1	1	9,578.05
13	0	0	1	0	1	1	1	1	0	9,538.41
14	0	0	0	1	0	1	1	0	1	9,501.96
15	0	0	0	1	1	1	0	1	0	9,496.47
16	0	0	0	0	0	1	1	1	1	9,478.44
17	0	0	0	1	1	0	1	1	0	9,222.13

[1] "1" indicates that an Operational Strategy is included in the Investment Alternative.

OS(9,18) and *OS*(11,21), which requires a $67,000 investment and generates an additional annual profit of $21,371, resulting in a positive AW of $11,533.35.

Once again, if the operational strategies with the highest AW values had been selected, without violating any constraints, the resulting set would have been comprised of *OS*(4,8), *OS*(5,11), *OS*(9,18) and *OS*(11,21) –the fourth investment alternative in table 10-, which is suboptimal with an AW of $10,559.36.

3.4 Type 1 EMP Model to Increase Throughput and Minimize Marginal Energy Consumption Increase

Under this philosophy, as explained in section 3.1, a technology implementation will never result in overall energy consumption reduction since the operating parameter *Et* is not effected. In fact, the only way to reduce energy consumption is through reduction of material input, which is the case with the philosophy in the next section (Maintain Throughput and Minimize Energy Consumption).

Nevertheless, the implementation of a new operational strategy under this philosophy can have two types of energy-related impacts. First, when the strategy applies to an intermediate process, the company will require additional energy to process the increased material output through the subsequent process(es). Second, when the strategy applies a final process, the company will continue to consume the same amount of energy since the additional material output immediately becomes additional final product.

As a result, if we attempt to minimize marginal (or additional) energy consumption, we are limited to select only from operational strategies that apply to final processes. This approach does not necessarily maximize throughput. On the other hand, if we attempt to maximize total throughput, the enterprise will most likely end up consuming additional energy. In other words, this philosophy involves two conflicting objectives functions: minimize ΔE and maximize ΔT.

Two different techniques will be utilized in this research to deal with this philosophy. First, we will let throughput increase as much as possible in the most efficient way from an energy-consumption viewpoint. Basically, this technique, whose model formulation is presented in the next section, establishes a trade-off whereby a controlled marginal energy consumption is allowed.

The second technique, whose model formulation will be presented in section 4.4.2, will involve an efficient frontier to describe Pareto optimal solutions. Although this technique does not specify the best solution, it is very useful in multiple-objective decision making because it gives the ultimate decision maker many solutions to choose from.

3.4.1 Step 1: Estimating PBCs for Controlled-Marginal-Energy-Consumption Technique

If two or more operational strategies are applicable to two or more intermediate processes, a PBC (Power Benefit Coefficient) can be calculated to indicate which strategy is more attractive. A PBC measures, at an enterprise level, the total additional material throughput per unit of additional energy required at a given process; and it is calculated as

$$PBC_j = \Delta T_j \big/ \Delta E_j \tag{46}$$

where ΔT_j represents the additional material throughput generated by an additional ton of material output out of the j^{th} process, and is calculated as

$$\Delta T_j = \sum_{i=1}^{n} \left(\prod_{h,k} r_{h,k,i} \prod_{k} Y_k \right) \tag{47}$$

while ΔE_j is the additional energy required by the enterprise to process an additional ton of material output out of the j^{th} process, and is expressed as

$$\Delta E_j = \sum_{h=1}^{m} \left(Et_h \prod_{h,k} r_{h,k,0} \prod_{k} Y_k \right) \tag{48}$$

It must be noted that ΔT_j assumes that the increase of throughput for all final products is equally desirable –i.e., an extra ton of final product *A* is valued the same as an extra ton of final product *B*.- It is possible to assign desirability indexes to the different types of final products, but these user-driven indexes will not be included in this research.

As is the case with EBC values, intermediate-process PBC values may change as new operational strategies are selected. As a matter of fact, the introduction of a strategy makes all "upstream" operational strategies more attractive, although it does not impact the desirability of all "downstream" operational strategies.

3.4.2 Step 2: Operational Strategy Selection for Controlled-Marginal-Energy-Consumption Technique

The decision variable is the same binary variable used for the previous profit-driven EMP models: $OS(j,a)$. Moreover, upper-bound, capacity and budget constraints still apply since process yield cannot exceed 100%, no process can handle material above its capacity levels, and an available company budget cannot be surpassed, respectively.

The objective function for final-process strategies can be expressed as maximize

$$\sum_{a=1}^{q}\left(\Delta P_{j(a)} OS(j,a)\right) \tag{49}$$

since for all final-process strategies ΔE is equal to zero and their $\Delta P_{j(a)}$ is equivalent to ΔT. In turn, the objective function for intermediate-process strategies with the same PBC is maximize

$$\sum_{a=1}^{q}\left(\Delta T_j \; \Delta P_{j(a)} \; OS(j,a)\right) \tag{50}$$

given that $(\Delta T_j \; \Delta P_{j(a)})$ is equivalent to ΔT.

3.4.3 Model Statement

$$\text{Maximize} \begin{cases} \sum_{a=1}^{q}\left(\Delta P_{j(a)} OS(j,a)\right) & \text{if } \sum_{k=1}^{m} P_{j,k} = 0 \\[2ex] \sum_{a=1}^{q}\left(\Delta T_j \; \Delta P_{j(a)} \; OS(j,a)\right) & \text{if } \sum_{k=1}^{m} P_{j,k} > 0 \end{cases}$$

Subject to

$$OS(j,a) = \begin{cases} 1 & \text{if the } a^{\text{th}} \text{ new operational strategy for the } j^{\text{th}} \text{ process has been chosen } (a=1,2,\ldots,q) \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{Y}_j \leq 1 \qquad \text{for } j=1,2,\ldots,m \quad \text{(Upper-bound)}$$

$$\sum_{k=1}^{m} P_{k,j} + \widetilde{P}_j \leq K_j \qquad \text{for } j=1,2,\ldots,m \quad \text{(Capacity)}$$

$$\sum_{a=1}^{q}\left(I_a \; OS(j,a)\right) \leq B \qquad \text{(Budget)}$$

3.5 Type 1 EMP Model to Maintain Throughput and Minimize Total Energy Consumption

3.5.1 Step 1: Estimating PBCs

As indicated previously, when operational strategies are implemented under this philosophy, energy consumption is reduced because of the decreased amount of material input needed. In fact, if a strategy has the potential of generating additional ΔP material output from a process, the process will end up reducing its material input by $\Delta P / \hat{Y}$, where $\hat{Y}$ represents the improved yield. This material input reduction, in turn, saves both the energy it accumulated through the previous process plus the energy that it would have been spent at this process. These two savings form the power benefit (PB), which is calculated as

$$PB_j = (PW_j + Et_j)\frac{\Delta P_j}{\hat{Y}_j} = \frac{(PW_j + Et_j)\Delta P_j}{Y_j + \dfrac{\Delta P_j}{\sum_{k=0}^{m} P_{k,j}}} \tag{51}$$

after replacing Eq. (22); where PW_j represents the total accumulated energy per ton of product arriving at j^{th} process, and is calculated as

$$PW_j = \sum_{k=1}^{m} (PW_k + Et_k)\frac{F_{k,j}}{Y_k} \tag{52}$$

for $j=2,\ldots,m$, where $F_{k,j}$ is defined by Eq. (8) and PW_1 is equal to zero. However, when PB_j in Eq. (51) is divided by ΔP_j to calculate PBC_j, the result is still dependent upon ΔP_j since the marginal PBC value for each additional ton of material recovered monotonically declines.

Lastly, the introduction of a new operational strategy into an "upstream" process reduces the value of all "downstream" PBs since PW_j in Eq. (51) decreases. Moreover, due to the fact that the introduction of a strategy in a "downstream" process results in a reduction of "upstream" flow, "upstream" PBs also decrease since the potential material savings diminishes as the material input P_l diminishes.

3.5.2 Step 2: Operational Strategy Selection

The objective function is to minimize total energy consumption, which is equivalent to maximize total energy savings or

$$\sum_{a=1}^{q}\left(PB_j\, OS(j,a)\right) \tag{53}$$

3.5.3 Model Statement

Maximize $\displaystyle\sum_{a=1}^{q}\left(PB_j\, OS(j,a)\right)$

Subject to

$$OS(j,a) = \begin{cases} 1 & \text{if the } a^{\text{th}} \text{ new operational strategy for the } j^{\text{th}} \text{ process has been chosen} \\ 0 & \text{otherwise} \end{cases}$$

$$\hat{Y}_j \leq 1 \qquad \text{for } j=1,2,\ldots,m \quad \text{(Upper-bound)}$$

$$\sum_{a=1}^{q}\left[I_a\, OS(j,a)\right] \leq B \qquad \text{(Budget)}$$

In the upper-bound constraint above, $\hat{Y}_j$ is calculated as specified in Eq. (22) with ΔP_j representing the amount of material that *would have been recovered* at the j^{th} process had the decision been made to increase throughput.

3.6 Type 2 EMP Model Considerations

Given the fact that Type 1 EMP models for each one of the four optimization philosophies have been already presented, this section will only address specific issues that must be taken into account when material feedback loops are to be considered.

3.6.1 Material Recovery Categories

In the Type 1 EMP models discussed so far, it is assumed that ΔP_j, and consequently $\Delta P_{j(a)}$, represent the amount of material loss that becomes acceptable-quality product automatically increasing the j^{th} process' yield - see Eq. (22), (32), (44), and (51). However, when dealing with Type 2 EMP models, ΔP_j can represent three different kinds of material recovery based on the material flows in figure 5:

1) Material loss is transformed into acceptable-quality product (A:ML →AQP) resulting in a yield increase.
2) Material loss is transformed into recycled product (B:ML→RP) without affecting the yield.
3) Recycled product is transformed into acceptable quality product (C:RP→AQP) resulting in a yield increase.

Accordingly, ΔP_j also affects $r_{j,k,i}$ values as presented in Eq. (54) through (58) below, where $\hat{r}_{j,k,i}$ represents the modified value of $r_{j,k,i}$, $r_{j,k,0}$ corresponds to a forward (acceptable-quality) material flow if $j < k$, and $r_{j,k,0}$ corresponds to a feedback (recycled) material flow if $j \geq k$.

A:ML →AQP

$$\hat{r}_{j,k,0} = \begin{cases} r_{j,k,0} & \text{if } j \geq k \\ r_{j,k,0}\dfrac{\hat{Y}_j}{Y_j} & \text{if } j < k \end{cases} \tag{54}$$

$$\hat{r}_{j,0,i} = r_{j,0,i}\frac{\hat{Y}_j}{Y_j} \tag{55}$$

B:ML→RP

$$\hat{r}_{j,k,0} = \begin{cases} r_{j,k,0} & \text{if } j < k \\ 0 & \text{if } r_{j,k,0} = 0 \text{ and } j \geq k \\ r_{j,k,0} + \dfrac{\Delta P_j}{\sum_{l=0}^{m} P_{l,j}} & \text{if } r_{j,k,0} \neq 0 \text{ and } j \geq k \end{cases} \quad (56)$$

$r_{j,0,i}$ does not change

C:RP→AQP

$$\hat{r}_{j,k,0} = \begin{cases} r_{j,k,0}\dfrac{\hat{Y}_j}{Y_j} & \text{if } j < k \\ 0 & \text{if } r_{j,k,0} = 0 \text{ and } j \geq k \\ r_{j,k,0} - \dfrac{\Delta P_j}{\sum_{l=0}^{m} P_{l,j}} & \text{if } r_{j,k,0} \neq 0 \text{ and } j \geq k \end{cases} \quad (57)$$

$$\hat{r}_{j,0,i} = r_{j,0,i}\frac{\hat{Y}_j}{Y_j} \quad (58)$$

where Y_j and $\hat{Y}_j$ are calculated using Eq. (11) and (22), respectively.

Therefore, for Type 2 EMP models, the variables ΔP_j and $\Delta P_{j(a)}$ will have the superscripts *A*, *B* or *C* to indicate the kind of material recovery as explained above. For instance, the variables will be ΔP_j^A and $\Delta P_{j(a)}^A$ if they represent an A:ML →AQP category of material recovery.

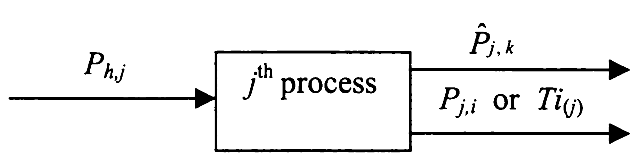
It is also important to note that, in this research, ΔP_j will be allocated so that the participation of intermediate products and final products relative to the total acceptable-quality product coming out of a given process remains constant. In other words, if 70% of the acceptable-quality product out of the j^{th} process is intermediate product and 30% is final product, 70% of ΔP_j will become additional intermediate product and 30% additional final product.

3.6.2 Rebalancing Material Flows to Maintain Final-Product Throughputs

As indicated previously, the material flows in the EM model must be rebalanced after the selection of a new strategy when an optimization philosophy seeks to maintain throughput.

The rebalancing of individual processes will be done under the following guidelines:

a) No adjustment is needed for a process downstream of the improved process.
b) If a process is upstream of the improved process and has a single acceptable-quality output, its material input must be recalculated based on its diminished intermediate material output and the process' yield.
c) If a process is upstream of the improved process and has two or more acceptable-quality outputs, intermediate and/or final products, both its material input and its acceptable-quality $r_{j,k,i}$ values must be recalculated as presented below, where $\hat{P}_{j,k}$ represents the diminished output.



$$\hat{P}_{h,j} = \frac{\hat{P}_{j,k} + P_{j,l}}{Y_j},$$

$$\hat{r}_{j,k,0} = {\hat{P}_{j,k}}\Big/{\hat{P}_{h,j}}, \text{ and}$$

$$\hat{r}_{j,l,0} = {P_{j,l}}\Big/{\hat{P}_{h,j}}.$$

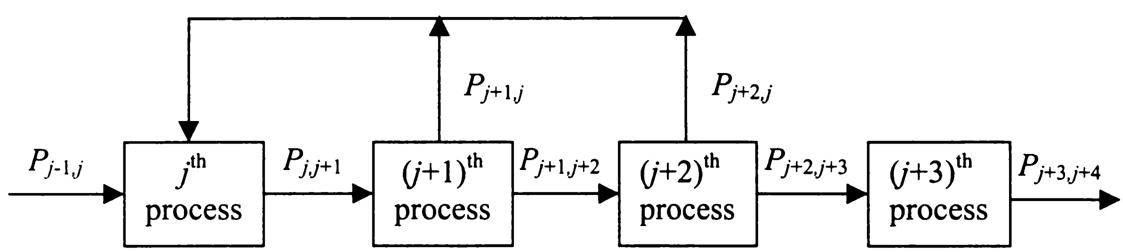
Furthermore, due to the presence of feedback loops, there are three possible material rebalancing scenarios:

1. The improved process is upstream of the entry point of the loop: no adjustment required.

2. The improved process is part of the loop.

2.1. The improved process is the entry point of the loop: no adjustment needed.

2.2. The improved process is inside the loop: downstream part of the loop, including feedback flows that originate downstream of the improved process, does not change. However, upstream part of the loop, including feedback flows that originate upstream of the improved process, must be rebalanced. When calculating intermediate material input at the loop's entry point, feedback material must be considered as shown in the example below, where the $(j+1)^{\text{th}}$ process is improved by ΔP^{A}_{j+1}.



$$\hat{P}_{j,\,j+1} = {P_{j+1,\,j+2}}\Big/{\hat{Y}_{j+1}},$$

$$\hat{P}_{j+1,\,j} = r_{j+1,\,j,0}\,\hat{P}_{j,\,j+1}\text{, and}$$

$$\hat{P}_{j-1,\,j} = \frac{\hat{P}_{j,\,j+1}}{Y_j} - \hat{P}_{j+1,\,j} - P_{j+2,\,j}$$

2.3. The improved process is the exit point of the loop: the entire loop must be rebalanced. Once again, when calculating intermediate material input at the loop's entry point, feedback material must be considered as presented above.

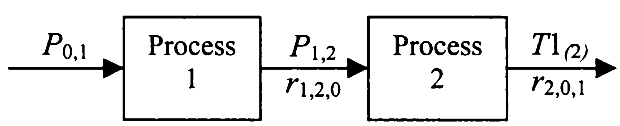
3. The improved process is downstream of the exit point of the loop: the entire loop must be rebalanced since its intermediate material output at the loop's exit point is reduced.

3.6.3 Calculating Benefit Coefficients and Capacity Constraints

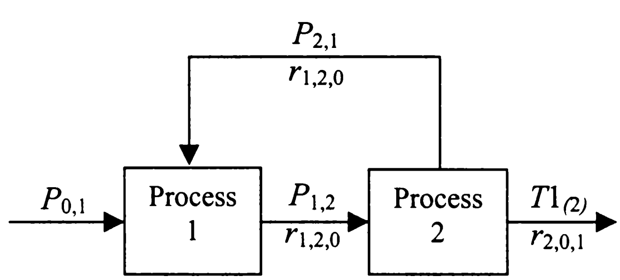
This section illustrates the impact that the introduction of material feedback loops has on the calculation of benefit coefficients, more specifically EBs, and the generation of the capacity constraints.

If Eq. (27) is used to estimate EB_1 in the 2-process Type 1 EM model below under the increase-throughput, maximize-profit philosophy, it can be written as

$$EB_1 = (R_1 - C'_2)\,\Delta P_1\, r_{2,0,1}$$



If a material feedback flow is added to the process, as portrayed below, EB_1 will asymptotically approach its equilibrium state's value as exemplified next.



Before the consideration of the new operational strategy,

$$T1_{(2)} = P_{1,2}\, r_{2,0,1} = (P_{0,1} + P_{2,1})\, r_{1,2,0}\, r_{2,0,1}$$

If the implementation of a strategy can provide a ΔP_1^A,

$$\hat{T}1_{(2)} = \hat{P}_{1,2}\, r_{2,0,1} = \left(P_{1,2} + \Delta P_1^A\right) r_{2,0,1}$$

after the first production cycle; but since

$$\hat{P}_{2,1} = \hat{P}_{1,2}\, r_{2,1,0}$$

then

$$\hat{T}1_{(2)} = (P_{0,1} + \hat{P}_{2,1})\, \hat{r}_{1,2,0}\, r_{2,0,1}$$

after the second production cycle. In other words, the value of $T1_{(2)}$ has to be rebalanced. Moreover, the value of C'_2 also has to be rebalanced even after production flows have achieved steady state.

Likewise, a capacity constraint can only be reformulated until after the system has been rebalanced. Moreover, its complexity level significantly rises in the presence of feedback loops. For instance, in Eq. (41)

$$\widetilde{P}_4 = \Delta P_3 + \hat{Y}_3 \Delta P_2 + \hat{Y}_3 \hat{Y}_2 \Delta P_1$$

for the hypothetical Type 1 production process in figure 6 without feedback loops. If the feedback loops in figure 6 are included,

$$\widetilde{P}_4 = \Delta P_3 + \hat{Y}_3 \Delta P_2 + \hat{Y}_3 \hat{Y}_2 \Delta P_1 + \hat{Y}_3 \hat{Y}_2 \hat{Y}_1 (\Delta P_{3,1} + \Delta P_{5,1} + \Delta P_{7,1}) \qquad (59)$$

where

$$\hat{Y}_1 = 0.98 + \frac{\Delta P_1}{10{,}792}$$

$$\hat{Y}_2 = 0.99 + \frac{\Delta P_2}{10{,}577}$$

$$\hat{Y}_3 = 0.98 + \frac{\Delta P_3}{10{,}471}$$

This yields a non-linear formula for $\widetilde{P}_j$ variables with a cumbersome algebraic form. Furthermore, the calculation of their values can only be accomplished once the feedback flows are determined (represented by the last term in Eq. (59)), and the feedback flows are themselves a non-linear function of the total quantity of material recovered from processes 1 through 7 as represented below for the feedback-flow terms in Eq. (59),

$$\Delta P_{3,1} = f(\Delta P_1, \Delta P_2, \ldots, \Delta P_7)$$

$$\Delta P_{5,1} = f(\Delta P_1, \Delta P_2, \ldots, \Delta P_7)$$

$$\Delta P_{7,1} = f(\Delta P_1, \Delta P_2, \ldots, \Delta P_7)$$

CHAPTER 4

SOLUTION METHODOLOGY FOR EMP MODELS

4.1 Problem Formulation Considerations

A problem is said to be mathematically defined in *standard form* if it is expressed as

$$\begin{array}{lll} \text{Max/Min} & f(x_1, x_2, \ldots, x_n) & \\ \text{subject to} & & \\ & g_i(x_1, x_2, \ldots, x_n) \{\leq = \geq\} b_i & i = 1,2,\ldots,m \\ & x_j \geq 0 & j = 1,2,\ldots,n \end{array}$$

where f is the objective function, g_i the constraint functions, and x_j the decision variables.

In other words, the purpose of a mathematical programming problem is to determine the values of the decision variables that optimize the objective function without violating the constraints.

The EMP models presented in chapter 3 can be generalized in standard form as follows:

- The decision variables $OS(j,a)$ are limited to the values zero and one.
- The objective function is linear and defined as

  $$\text{Max} \quad \sum_{a=1}^{q} \left(U_{j,a}\, OS(j,a) \right)$$

  where $U_{j,a}$ is a coefficient representing AW_a, $\Delta P_{j(a)}$, $\Delta T_j\ \Delta P_{j(a)}$, or PB_j depending upon the optimization philosophy.
- The budget constraint is linear and defined as

  $$\sum_{a=1}^{q} \left(I_a\, OS(j,a) \right) \leq B$$

- The upper-bound constraints are linear and defined as

$$Y_j + \frac{1}{\sum_{k=0}^{m} P_{k,j}} \sum_{a=1}^{q} \left(\Delta P_{j(a)}\, OS(j,a)\right) \le 1 \qquad \text{for } j=1,2,\ldots,m$$

- The capacity constraints, which apply only to those philosophies that allow throughput increase, are nonlinear and defined as

$$g_j\left(OS(j,a)\,|_{\,a=1,\ldots,q}\right) \le K_j \qquad \text{for } j=1,2,\ldots,m$$

Furthermore, mathematical programming problems can be classified in five different ways as follows (Pfaffenberger and Walker, 1976):

1. The functional relationships in the problem may be known (deterministic) or uncertain (stochastic).
2. The objective and constraint functions may be linear, or at least one function in the set may be nonlinear.
3. The functions may be differentiable or non-differentiable.
4. The decision variables may be continuous or restricted to integer values.
5. The optimization may take place at a fixed point in time (static) or during an interval of time (dynamic).

Accordingly, the EMP models are static, deterministic, nonlinear binary integer problems for those philosophies that allow throughput increase; and static, deterministic, linear binary integer problems when the throughput is maintained. Nevertheless, if the capacity constraints are ignored, the EMP models become the *multi-dimensional zero-one Knapsack type problem* (Lau, 1986), whose standard form is

$$\text{Max} \quad \sum_{j=1}^{n} c_j x_j$$

$$\text{Subject to} \quad \sum_{j=1}^{n} a_{ij} x_j \le b_i \qquad \text{for } i=1,2,\ldots,m$$

$$x_j = 0,\, 1 \qquad \text{for } j=1,2,\ldots,n$$

Furthermore, if the upper-bound constraints are ignored, the EMP models become the extensively studied *zero-one Knapsack problem* (Winston, 1987), whose standard form is

$$\text{Max} \quad \sum_{j=1}^{n} c_j x_j$$

$$\text{Subject to} \quad \sum_{j=1}^{n} a_j x_j \leq b$$

$$x_j = 0, 1 \qquad \text{for } j=1,2,\ldots,n$$

4.2 Numerical Solution Considerations

A question that arises as the issue of determining how a mathematical programming model is to be solved numerically is whether a heuristic or an optimization approach is necessary (Moder and Elmaghraby, 1978).

A heuristic algorithm is "a technique that seeks near-optimal solutions at a reasonable computational cost without being able to guarantee either feasibility or optimality, or even in many cases to state how close to optimality a particular feasible solution is" (Reeves, 1993). In other words, a heuristic is a method for obtaining "good" solutions while minimizing the number of feasible solutions evaluated - although it provides assurance of neither obtaining the optimal solution nor even ultimately obtaining any solution. To obtain a near-optimal solution, a heuristic normally exploits a specific characteristic of the problem at hand; and its ability to find the global optimum is heavily dependent on the number of local optima. Consequently, local search methods -also known as meta-algorithms or metaheuristics because of their general search principles organized in a general search strategy- such as Tabu search, Simulated Annealing and Genetic algorithms are often applied once the heuristic solutions are achieved in an attempt to "escape" local optima to improve the heuristic solutions (Aarts and Lenstra, 1997).

An optimization algorithm, on the other hand, is a method that provides a mathematical guarantee of convergence to an optimum in a finite number of steps. However, because the rigorous requirements of the optimization methods sometimes result in computationally inefficient algorithms or the simplification of the model in order to obtain an optimal solution, a heuristic approach is considered to be a better option in many cases since it represents an approximate solution to an exact formulation of the problem (Kreher and Stinson, 1999).

Six main approaches have been used to solve integer programming problems (Salkin and Mathur, 1989): cutting plane techniques that deduce supplementary inequalities or *cuts* from the integrality and constraint requirements, enumerative methods that list either explicitly or implicitly all possible solution candidates, dynamic programming with the shortest path concept, strong valid inequalities with the idea of looking for the most violated inequality, Lagrangian duality that moves complicating constraints into the objective function, and column generation algorithms with the decomposition concept.

In turn, four different optimization methods exist for the zero-one Knapsack program (Korte and Vygen, 2000): dynamic programming, which is inefficient when the value of the right-hand-side constant is large; network approaches, which are normally inefficient because of the enormous size of the resulting shortest-route network; generalized Lagrangean methods, which are computationally satisfactory when approximate solutions are required; and implicit enumeration methods, which are more efficient under a variety of circumstances.

Within implicit enumeration methods, which decompose the feasible area into subsets and discards those subsets that cannot possibly contain the optimal solution through the use of upper and lower bounds for each subset, the branch-and-bound method is the most popular and successful computational approach to Knapsack type problems today. The branch and bound method, which was first suggested by Land and Doig (1960), has four fundamental elements (Eiselt and Sandblom, 2000): separation (partition) into subproblems, relaxation (upper bounding), fathoming of subproblems (lower bounding), and selection of subproblems (branching).

Partitioning is the process of dividing the set of feasible solutions into smaller sets with the property that any optimal solution must be in at least one of the subsets. Next, upper bounds are calculated for each subset by solving some sort of relaxation. After that, any subproblem can be discarded or fathomed when the optimum is found, when it is determined that its subset does not contain a solution that can improve on the best current solution, or when it is unfeasible. Finally, branching contends with the decision as to what to do next. Basically, at any stage, the options available are (a) to solve the relaxation for a subproblem, (b) to select a specific subproblem and partition it, and (c) to attempt to achieve a better lower bound to achieve further fathoming.

Regardless of the approach used (heuristic or optimization), the numerical solution involves an algorithm or a set of mathematical rules for solving a particular class of problem or model. Although different types of algorithms have been used, practitioners of operations research have favored finite improvement algorithms, whose basic concept is to move from a potential solution to a better one at each step as follows (Solow, 1984):

Step 0. Initialization: select an initial solution.

Step 1. Test for Optimality/Adequacy: perform a test to determine whether the current solution provides the optimum objective value (optimization) or an adequate objective value (heuristic). If so, stop. Otherwise, go to step 2.

Step 2. Moving: find a path in the feasible region that leads to another solution that provides a better objective value. Move along this path to the new solution and determine the new values of the variables. Return to step 1.

The key to a finite improvement algorithm lies in the ability to formulate a test for optimality or adequacy such that, if the current solution is neither optimal nor acceptable, then that fact will point out how to select a new solution with a strictly better objective value. When dealing with integer problems, optimality and adequacy are normally determined based on a bounding technique as explained below (Wolsey, 1998).

First, every feasible solution provides a lower or primal bound $\underline{Z}$ such that $\underline{Z} \leq Z$, where Z represents the optimal (maximum) solution. Second, relaxations provide upper or dual bounds $\overline{Z}$ such that $\overline{Z} \geq Z$. A relaxation means replacing a difficult optimization problem by a simpler optimization problem whose optimal solution is at least as large as Z. There are two obvious possibilities to relax a problem: enlarge the set of feasible solutions so that one optimizes over a larger set, or replace the objective function by a function that has the same or larger values everywhere. For instance, the most natural, and historically the first, relaxation of the zero-one Knapsack problem is the linear programming relaxation where the constraint $x_j \in \{0,1\}$ is replaced by $0 \leq x_j \leq 1$. A zero-one Knapsack algorithm will then find a decreasing sequence

$$\overline{Z}_1 > \overline{Z}_2 > \ldots > \overline{Z}_s \geq Z$$

of upper bounds, and an increasing sequence

$$\underline{Z}_1 < \underline{Z}_2 < \ldots < \underline{Z}_t \leq Z$$

of lower bounds, and stop when

$$\overline{Z}_s - \underline{Z}_t \leq \varepsilon,$$

where ε is equal to zero for the optimal solution and some suitably chosen small nonnegative value for the heuristic solution.

4.3 Numerical Methods to Maximize Profit

In the past, most of the work done in discrete optimization has been concerned with linear integer programming. In other words, the area of nonlinear integer programming has been, as noted by Gisvold and Moe (1972), "reluctant to part from the relatively safe harbors of linear programming." This fact is understandable if one considers that algorithms for the solution of linear integer problems are hard to be solved in a reasonable amount of time due to significant computational difficulties that become even worse for nonlinear integer programming.

Consequently, nonlinearities in integer programming are normally handled by the use of methods involving piecewise linear approximations, or involving the transformation of a nonlinear function into a polynomial function of binary variables. Techniques have also been formulated for solving nonlinear integer problems directly by involving separable functions with certain monotonicity characteristics. Some other methods have successfully applied penalty functions to the objective function when integer variables take on non-integer values (Glover, 1975).

However, some researchers have entered the nonlinear arena by handling specific classes of nonlinear problems; i.e., special cases of f and g_i in the standard form presented above (Cooper, 1981). For example, classes such as convex separable, concave, polynomial, quadratic, parabolic, and hyperbolic problems have been analyzed. In fact, Agrawal (1975) was the first to apply the branch and bound method to problems having a quadratic function and linear constraints. This work was followed by Gupta (1980), Cabot and Erenguc (1986), West (1988), and Lee (1989); but the success of the branch and bound algorithms has been found to be largely dependent on the ability to solve the continuous problems and the number of subproblems to be solved, which is a serious consideration if the function evaluations are complex. Nowadays, strategies such as parallel processing, which allows the simultaneous exploration of nodes and branches, expert systems and artificial intelligence are being investigated.

In addition to the complexities previously discussed, the nonlinear constraints in the EMP models are process specific and their algebraic form is difficult to generate, although the feasibility of any solution can be checked through the EM model since all process constraints (upper-bound and capacity) are embedded in it. Moreover, the values of the objective function coefficients, which are determined by the benefit coefficients discussed in chapter 3, are a function of the solution being considered.

As a result, the numerical method described by the flowchart in figure 8 will be used for profit maximization. This method first solves the non-relaxed problem using a heuristic and conducts a local search in an attempt to improve the heuristic solution. The method then finds a second solution through either a heuristic for the multi-dimensional zero-one knapsack type problem resulting from the relaxation of the capacity constraints, or a branch-and-bound algorithm for the zero-one knapsack resulting from the elimination of the upper-bound constraints and the relaxation of the capacity constraints. Since the solution obtained through the last two techniques may be not be feasible due to the capacity constraint relaxation, the method performs a local search to find a feasible solution. The final solution will then be either the best heuristic solution obtained or the feasible branch-and-bound solution.

It must be noted that the second step of the method can be achieved through the use of maximum-value EBCs as explained in section 3.2.2 for the first step of figure 4. Furthermore, to check in step 3 whether any upper-bound constraint can be potentially exceeded, all the decision variables $OS(j,a)$ will be made equal to 1 and replaced in the equations

$$Y_j + \frac{1}{\sum_{k=0}^{m} P_{k,j}} \sum_{a=1}^{q} \left(\Delta P_{j(a)}\, OS(j,a) \right) \qquad \text{for } j=1,2,\ldots,m$$

If no equation exceeds the value 1, all upper-bound constraints are redundant and can be discarded.

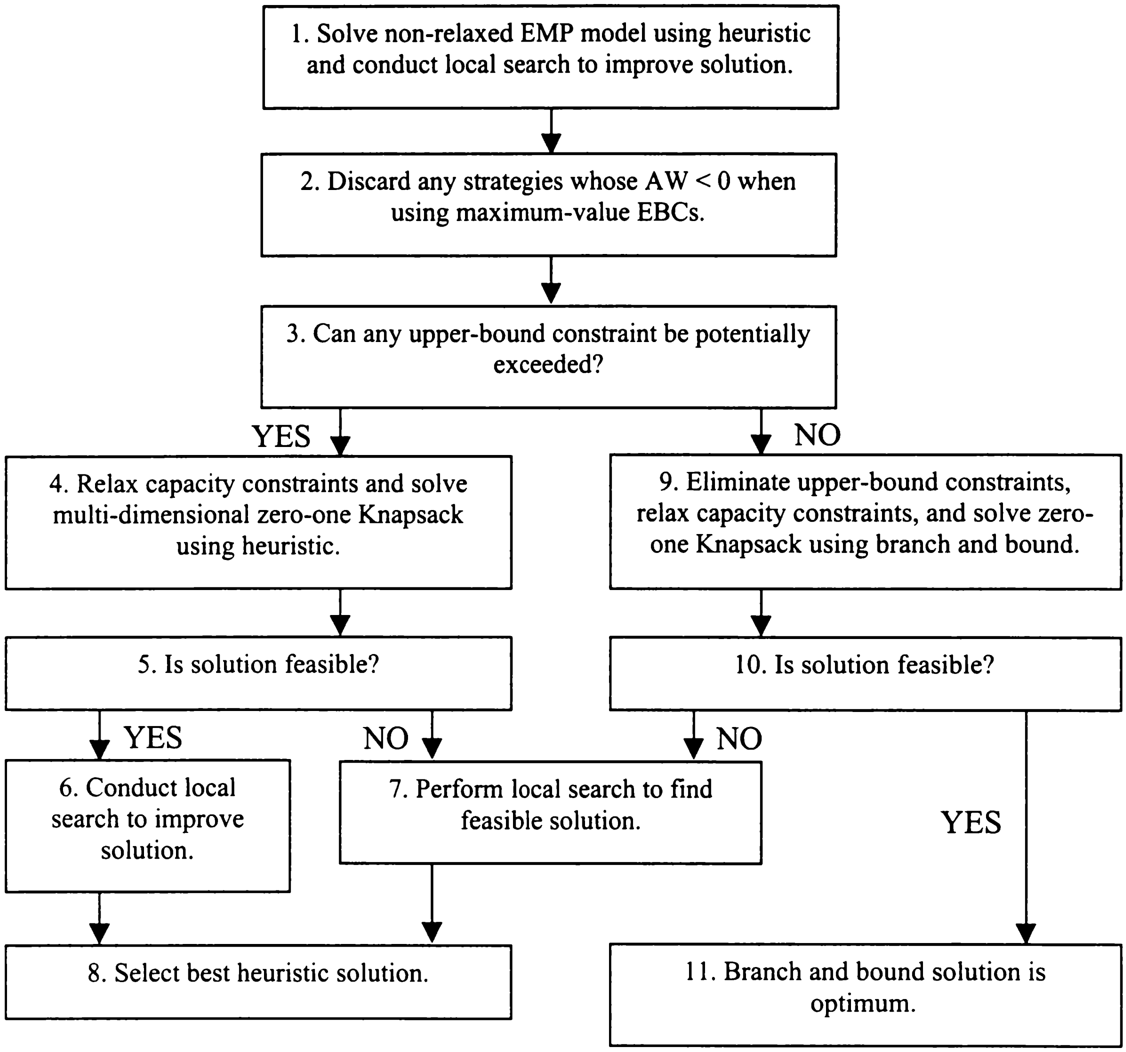
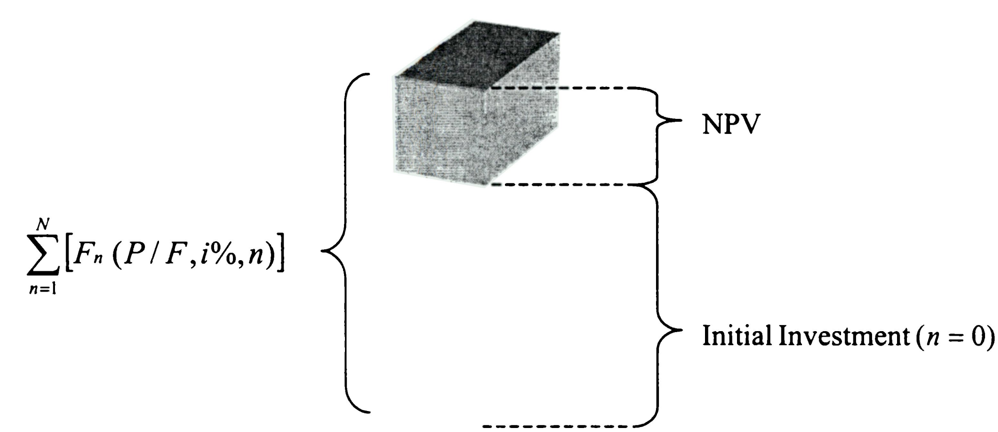


Figure 8. Numerical Methods for Profit Maximization.

4.3.1 Non-Relaxed EMP Model Heuristic Method

Although the economic theory method previously illustrated guarantees an optimum, the profit-maximization EMP models do not lend themselves to its lengthy application (more than 1 billion and 1 trillion investment alternatives have to be considered when the number of operational strategies is greater than or equal to 30 and 40, respectively), a more efficient computational procedure is needed.

As indicated in section 1.1, the selection of projects with the highest economic worth measure without exceeding the budget does not guarantee an optimum allocation even when equivalent worth measures are utilized (Park and Sharp-Bette, 1990). The reason is that equivalent worth methods such as NPV and AW measure the surplus in an investment over and above the investment at the MARR. In other words, NPV and AW values are independent from initial investment values as presented below for a positive NPV (F_n represents the net cash flow in the n^{th} time period).



Nevertheless, given the fact that an optimum solution for the EMP models maximizes the combined AW for a given B, i.e. the value $\sum_{a=1}^{q}\left(AW_a\; OS(j,a)\right)/B$; an "index" that measures the relationship between the strategy's AW and its required investment will overcome the limitation mentioned above. Simply stated, the best strategies from an economic point of view are those with the highest AW_a/I_a or annual-worth-per-dollar ratios. In other words, if $\dfrac{AW_A}{I_A} > \dfrac{AW_B}{I_B}$, strategy A is economically preferred to strategy B. This can be easily seen when $I = I_A = I_B$ is replaced in $\frac{AW_A}{I_A} > \frac{AW_B}{I_B}$, which results in $AW_A > AW_B$ confirming that strategy A should be preferred over B.

Consequently, an algorithm that uses the AW_a/I_a index as the criterion to select operational strategies will yield a better solution while significantly lessening the computational effort from 2^q to q maximum potential iterations. However, such an algorithm cannot guarantee optimality unless the first strategy rejected when choosing strategies according to the decreasing value of their AW_a/I_a ratios is so because the budget was completely exhausted after the selection of the previous strategy. The heuristic algorithm to select operational strategies that maximize profit under philosophies 1 and 2 is presented in figure 9.

The next important consideration is to estimate how close a given heuristic solution is to the optimum. Out of the three possible approaches to answer this question (*a priori* guarantee that the heuristic will provide a solution for the problem within ε or α% of optimal, *a priori* guarantee that the heuristic will on average produce a solution for the class of problems within α% of optimal, and *a posteriori* evaluation of the solution), *a posteriori* evaluation is normally preferred because of the higher degree of accuracy involved (Wolsey, 1998). This approach normally entails a bounding method that provides a close estimate of the value of the maximum (minimum) that, together with the value of the heuristic solution obtained, makes it possible to establish a valid upper (lower) bound to the error by which the heuristically obtained solution is affected.

If the model statement for the EMP model under philosophies 1 and 2 is expressed as

$$\text{Maximize} \quad \sum_{a=1}^{q}\left(AW_a\, OS(j,a)\right)$$

$$\text{Subject to} \quad \sum_{a=1}^{q}\left(I_a\, OS(j,a)\right) \le B$$

$$OS(j,a) \in \{0,1\} \qquad \text{for } a=1,2,\ldots,q$$

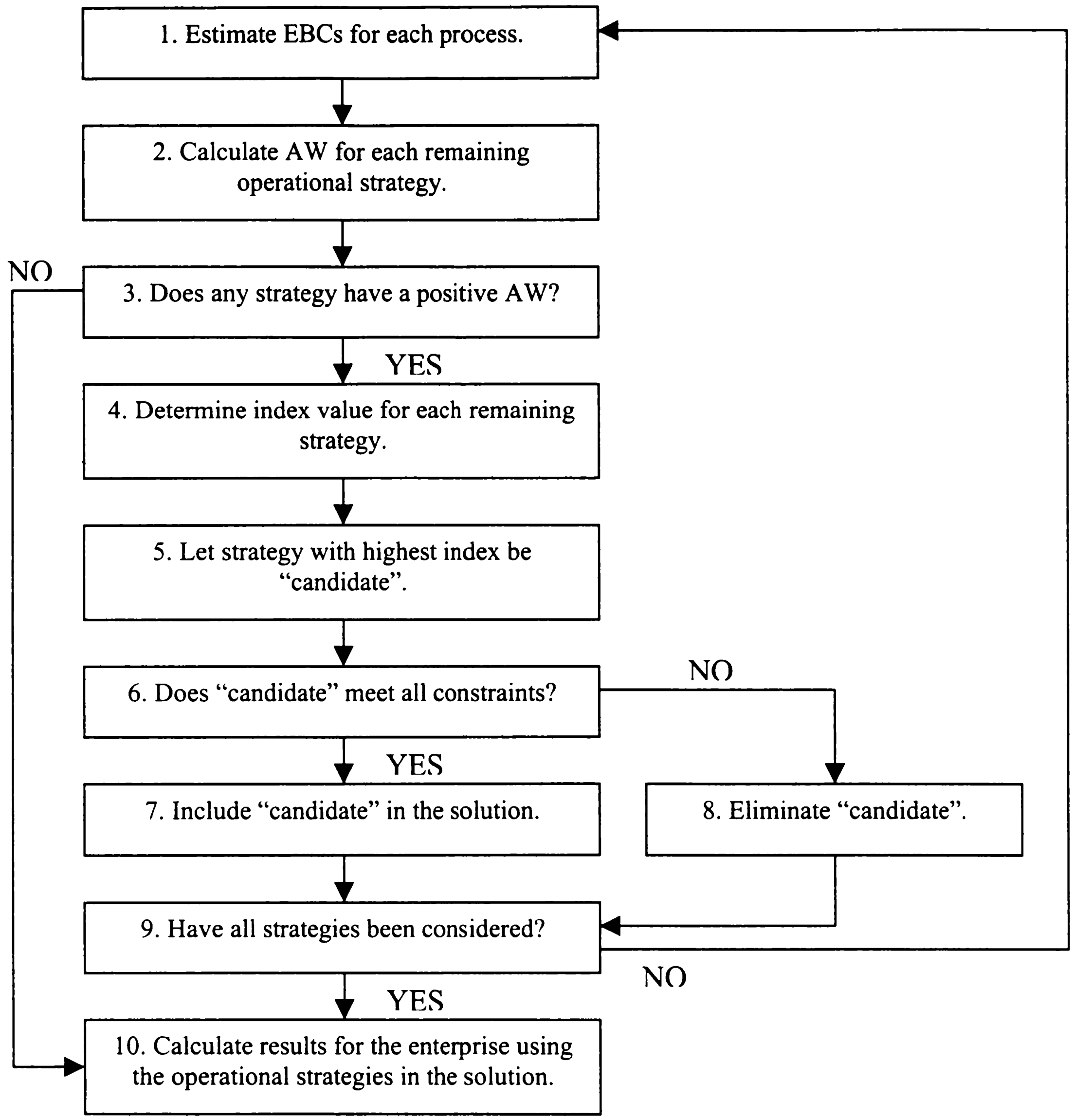


Figure 9. Heuristic Algorithm for Maximizing Profit.

after finding the heuristic solution, which obviously does not violate upper-bound nor capacity constraints due to the feasibility test in step 6 of figure 8; the following theorem can be used to estimate an upper bound for the optimum value of the objective function.

Theorem 1 (Dantzig, 1957). If the objects in the zero-one Knapsack problem are ordered according to the decreasing values of the profits per unit weight, i.e. so that

$$AW_1/I_1 \geq AW_2/I_2 \geq \ldots \geq AW_q/I_q$$

and

$$s = \text{largest integer for which} \sum_{a=1}^{s} I_a \leq B,$$

the optimal solution to the associated continuous problem (i.e., the linear programming relaxation) is

$$OS(j,a) = 1 \qquad \text{for } a=1,2,\ldots,s$$

$$OS(j,a) = 0 \qquad \text{for } a=s+2,\ldots,q$$

$$OS(j,s+1) = \frac{\left(B - \sum_{a=1}^{s} I_a\right)}{I_{s+1}}$$

Corollary 1. (Dantzig, 1957). The value

$$UB_1 = \sum_{a=1}^{s} AW_a + \left\lfloor \left(B - \sum_{a=1}^{s} I_a\right) \frac{AW_{s+1}}{I_{s+1}} \right\rfloor \tag{60}$$

is an upper bound of the solution to the 0-1 Knapsack problem, where $\lfloor z \rfloor$ is the greatest integer less than or equal to z.

Out of the various improvements on Dantzig's upper bound that have been proposed; the one by Hudson (1977), which follows, deserves special attention.

Theorem 2. Let

$$s = \text{largest integer for which} \sum_{a=1}^{s} I_a \leq B$$

$$B_1 = \sum_{a=1}^{s} AW_a + \left\lfloor \left(B - \sum_{a=1}^{s} I_a \right) \frac{AW_{s+2}}{I_{s+2}} \right\rfloor \tag{61}$$

$$s^* = \text{largest integer for which } \sum_{a=1}^{s^*} I_a \leq B - I_{s+1}$$

$$B_2 = AW_{s+1} + \sum_{a=1}^{s^*} AW_a + \left\lfloor \left(B - I_{s+1} - \sum_{a=1}^{s^*} I_a \right) \frac{AW_{s^*+1}}{I_{s^*+1}} \right\rfloor \tag{62}$$

Then

$$UB_2 = \max \{B_1, B_2\} \tag{63}$$

is an upper bound for the 0-1 Knapsack problem and

$$UB_2 \leq UB_1$$

4.3.1.1 Numerical Example for Increase-Throughput, Maximize-Profit Philosophy's Heuristic

In figure 10, four groups of information are presented in a worksheet format for each iteration of the heuristic in figure 8 when applied to the Type 1 hypothetical example in figure 6: *Processes* presents the updated yield and EBC values for each process; *Operational Strategies* shows the AW and index values for each operational strategy; *Constraints* displays the value for each one of the constraint variables if the strategy with the highest index, which appears in bold style and over a gray background, is considered for inclusion in the solution; and *Solution Set* keeps track of the strategies selected.

The first iteration, whose EBC values are the same as the initial ones in table 6, selects the 21st strategy. The second and third iterations choose the 11th and 12th strategies, respectively. However, the 12th strategy violates the capacity constraint for the 7th process and is eliminated ($\Delta\widetilde{P}_7$ cannot exceed 27 tons). The fourth and fifth iterations pick the 17th and 18th strategies, respectively. In turn, the sixth iteration attempts to elect the 8th strategy, but this strategy is eliminated because it infringes both the capacity constraint for the 7th process and the \$70,000 budget constraint. The seventh iteration selects the 7th strategy. Next, the eighth iteration chooses the 3rd strategy, but this strategy again breaches a capacity and the budget constraints. At this point, however, it is obvious that since no remaining strategy requires an investment below

FIRST ITERATION

PROCESSES		
PR	Y_j	EBC_j
1	0.980	50.838
2	0.990	69.533
3	0.980	85.238
4	0.970	100.245
5	0.960	119.034
6	0.930	135.842
7	0.920	156.350
8	0.990	165.000
9	0.970	132.598
10	0.950	153.500
11	0.940	175.000
12	0.910	145.000

OPER. STRATEGIES		
OS	AW_a	Index
1	-874.19	-0.050
2	-1,023.53	-0.055
3	1,239.82	0.089
4	540.12	0.034
5	2,104.84	0.084
6	-1,305.14	-0.034
7	1,720.72	0.138
8	4,257.07	0.164
9	-1,097.41	-0.051
10	-601.49	-0.045
11	2,782.76	0.265
12	2,417.72	0.220
13	-406.03	-0.031
14	-1,283.37	-0.112
15	-117.42	-0.007
16	-366.65	-0.035
17	2,399.02	0.200
18	3,916.92	0.182
19	-547.59	-0.038
20	-344.66	-0.018
21	**4,018.11**	**0.335**
22	-183.31	-0.009
23	-7.48	0.000

CONSTRAINTS			
j	$\hat{Y}_j$	$\bar{P}_j$	$\sum I_a$
1	0.980	0.0	
2	0.990	0.0	
3	0.980	0.0	
4	0.970	0.0	
5	0.960	0.0	
6	0.930	0.0	
7	0.920	0.0	
8	0.990	0.0	
9	0.970	0.0	
10	0.950	0.0	
11	0.954	0.0	
12	0.910	0.0	
		Total	12,000

SOLUTION SET = $OS(11,21)$

SECOND ITERATION

PROCESSES		
PR	Y_j	EBC_j
1	0.980	51.480
2	0.990	70.182
3	0.980	85.900
4	0.970	100.928
5	0.960	119.824
6	0.930	135.842
7	0.920	156.350
8	0.990	165.000
9	0.970	133.762
10	0.950	155.951
11	0.954	175.000
12	0.910	145.000

OPER. STRATEGIES		
OS	AW_a	Index
1	-839.48	-0.048
2	-990.74	-0.054
3	1,293.82	0.092
4	575.84	0.036
5	2,161.82	0.086
6	-1,256.11	-0.033
7	1,751.46	0.140
8	4,325.38	0.166
9	-1,071.45	-0.050
10	-585.78	-0.043
11	**2,815.96**	**0.268**
12	2,417.72	0.220
13	-406.03	-0.031
14	-1,283.37	-0.112
15	-117.42	-0.007
16	-366.65	-0.035
17	2,439.78	0.203
18	3,990.28	0.186
19	-531.29	-0.037
20	-295.63	-0.016
21		
22	-183.31	-0.009
23	-7.48	0.000

CONSTRAINTS			
j	$\hat{Y}_j$	$\bar{P}_j$	$\sum I_a$
1	0.980	0.0	
2	0.990	0.0	
3	0.980	0.0	
4	0.970	0.0	
5	0.965	0.0	
6	0.930	12.6	
7	0.920	11.7	
8	0.990	10.8	
9	0.970	29.4	
10	0.950	14.3	
11	0.954	13.5	
12	0.910	8.6	
		Total	22,500

SOLUTION SET = $OS(11,21)$
$OS(5,11)$

Figure 10. Philosophy 1 Non-Relaxed Heuristic's Numerical Example Worksheets.

THIRD ITERATION

PROCESSES		
PR	Y_j	EBC_j
1	0.980	51.994
2	0.990	70.701
3	0.980	86.430
4	0.970	101.474
5	0.965	119.824
6	0.930	135.842
7	0.920	156.350
8	0.990	165.000
9	0.970	133.762
10	0.950	155.951
11	0.954	175.000
12	0.910	145.000

OPER. STRATEGIES		
OS	AW_a	Index
1	-811.75	-0.046
2	-964.55	-0.052
3	1,336.96	0.095
4	604.37	0.038
5	2,207.34	0.088
6	-1,216.94	-0.032
7	1,776.02	0.142
8	4,379.95	0.168
9	-1,050.72	-0.049
10	-573.23	-0.042
11		
12	**2,417.72**	**0.220**
13	-406.03	-0.031
14	-1,283.37	-0.112
15	-117.42	-0.007
16	-366.65	-0.035
17	2,439.78	0.203
18	3,990.28	0.186
19	-531.29	-0.037
20	-295.63	-0.016
21		
22	-183.31	-0.009
23	-7.48	0.000

CONSTRAINTS			
j	$\hat{Y}_j$	$\tilde{P}_j$	$\sum I_a$
1	0.980	0.0	
2	0.990	0.0	
3	0.980	0.0	
4	0.970	0.0	
5	0.965	0.0	
6	0.944	12.6	
7	0.920	**44.9**	
8	0.990	41.3	
9	0.970	29.4	
10	0.950	14.3	
11	0.954	13.5	
12	0.910	8.6	
		Total	33,500

SOLUTION SET = $OS(11,21)$
$OS(5,11)$

FOURTH ITERATION

PROCESSES		
PR	Y_j	EBC_j
1	0.980	51.994
2	0.990	70.701
3	0.980	86.430
4	0.970	101.474
5	0.965	119.824
6	0.930	135.842
7	0.920	156.350
8	0.990	165.000
9	0.970	133.762
10	0.950	155.951
11	0.954	175.000
12	0.910	145.000

OPER. STRATEGIES		
OS	AW_a	Index
1	-811.75	-0.046
2	-964.55	-0.052
3	1,336.96	0.095
4	604.37	0.038
5	2,207.34	0.088
6	-1,216.94	-0.032
7	1,776.02	0.142
8	4,379.95	0.168
9	-1,050.72	-0.049
10	-573.23	-0.042
11		
12		
13	-406.03	-0.031
14	-1,283.37	-0.112
15	-117.42	-0.007
16	-366.65	-0.035
17	**2,439.78**	**0.203**
18	3,990.28	0.186
19	-531.29	-0.037
20	-295.63	-0.016
21		
22	-183.31	-0.009
23	-7.48	0.000

CONSTRAINTS			
j	$\hat{Y}_j$	$\tilde{P}_j$	$\sum I_a$
1	0.980	0.0	
2	0.990	0.0	
3	0.980	0.0	
4	0.970	0.0	
5	0.965	0.0	
6	0.930	12.6	
7	0.920	11.7	
8	0.990	10.8	
9	0.976	29.4	
10	0.950	31.9	
11	0.954	30.3	
12	0.910	19.1	
		Total	34,500

SOLUTION SET = $OS(11,21)$
$OS(5,11)$
$OS(9,17)$

Figure 10. Continued.

FIFTH ITERATION

PROCESSES		
PR	Y_j	EBC_j
1	0.980	52.474
2	0.990	71.186
3	0.980	86.925
4	0.970	101.984
5	0.965	120.412
6	0.930	135.842
7	0.920	156.350
8	0.990	165.000
9	0.976	133.762
10	0.950	155.951
11	0.954	175.000
12	0.910	145.000

OPER. STRATEGIES		
OS	AW_a	Index
1	-785.82	-0.045
2	-940.06	-0.051
3	1,377.30	0.098
4	631.05	0.039
5	2,249.91	0.090
6	-1,180.31	-0.031
7	1,798.98	0.144
8	4,430.98	0.170
9	-1,031.32	-0.048
10	-561.49	-0.042
11		
12		
13	-406.03	-0.031
14	-1,283.37	-0.112
15	-117.42	-0.007
16	-366.65	-0.035
17		
18	**3,990.28**	**0.186**
19	-531.29	-0.037
20	-295.63	-0.016
21		
22	-183.31	-0.009
23	-7.48	0.000

CONSTRAINTS			
j	$\hat{Y}_j$	$\tilde{P}_j$	$\sum I_a$
1	0.980	0.0	
2	0.990	0.0	
3	0.980	0.0	
4	0.970	0.0	
5	0.965	0.0	
6	0.930	12.6	
7	0.920	11.7	
8	0.990	10.8	
9	0.988	29.4	
10	0.950	63.5	
11	0.954	60.3	
12	0.910	38.1	
		Total	56,000

SOLUTION SET = $OS(11,21)$
$OS(5,11)$
$OS(9,17)$
$OS(9,18)$

SIXTH ITERATION

PROCESSES		
PR	Y_j	EBC_j
1	0.980	53.339
2	0.990	72.059
3	0.980	87.816
4	0.970	102.903
5	0.965	121.469
6	0.930	135.842
7	0.920	156.350
8	0.990	165.000
9	0.988	133.762
10	0.950	155.951
11	0.954	175.000
12	0.910	145.000

OPER. STRATEGIES		
OS	AW_a	Index
1	-739.14	-0.042
2	-895.98	-0.048
3	1,449.91	0.104
4	679.07	0.042
5	2,326.53	0.093
6	-1,114.38	-0.029
7	1,840.32	0.147
8	**4,522.84**	**0.174**
9	-996.42	-0.046
10	-540.37	-0.040
11		
12		
13	-406.03	-0.031
14	-1,283.37	-0.112
15	-117.42	-0.007
16	-366.65	-0.035
17		
18		
19	-531.29	-0.037
20	-295.63	-0.016
21		
22	-183.31	-0.009
23	-7.48	0.000

CONSTRAINTS			
j	$\hat{Y}_j$	$\tilde{P}_j$	$\sum I_a$
1	0.980	0.0	
2	0.990	0.0	
3	0.980	0.0	
4	0.981	0.0	
5	0.965	90.0	
6	0.930	38.7	
7	0.920	**36.0**	
8	0.990	33.1	
9	0.988	90.2	
10	0.950	93.5	
11	0.954	88.9	
12	0.910	56.1	
		Total	**82,000**

SOLUTION SET = $OS(11,21)$
$OS(5,11)$
$OS(9,17)$
$OS(9,18)$

Figure 10. Continued.

SEVENTH ITERATION

PROCESSES		
PR	Y_j	EBC_j
1	0.980	53.339
2	0.990	72.059
3	0.980	87.816
4	0.970	102.903
5	0.965	121.469
6	0.930	135.842
7	0.920	156.350
8	0.990	165.000
9	0.988	133.762
10	0.950	155.951
11	0.954	175.000
12	0.910	145.000

OPER. STRATEGIES		
OS	AW_a	Index
1	-739.14	-0.042
2	-895.98	-0.048
3	1,449.91	0.104
4	679.07	0.042
5	2,326.53	0.093
6	-1,114.38	-0.029
7	**1,840.32**	**0.147**
8		
9	-996.42	-0.046
10	-540.37	-0.040
11		
12		
13	-406.03	-0.031
14	-1,283.37	-0.112
15	-117.42	-0.007
16	-366.65	-0.035
17		
18		
19	-531.29	-0.037
20	-295.63	-0.016
21		
22	-183.31	-0.009
23	-7.48	0.000

CONSTRAINTS			
j	$\hat{Y}_j$	$\tilde{P}_j$	$\sum I_a$
1	0.980	0.0	
2	0.990	0.0	
3	0.980	0.0	
4	0.975	0.0	
5	0.965	40.5	
6	0.930	24.3	
7	0.920	22.6	
8	0.990	20.8	
9	0.988	56.8	
10	0.950	77.0	
11	0.954	73.2	
12	0.910	46.2	
		Total	68,500

SOLUTION SET = $OS(11,21)$
$OS(5,11)$
$OS(9,17)$
$OS(9,18)$
$OS(4,7)$

EIGHTH ITERATION

PROCESSES		
PR	Y_j	EBC_j
1	0.980	53.811
2	0.990	72.537
3	0.980	88.303
4	0.975	102.903
5	0.965	121.469
6	0.930	135.842
7	0.920	156.350
8	0.990	165.000
9	0.988	133.762
10	0.950	155.951
11	0.954	175.000
12	0.910	145.000

OPER. STRATEGIES		
OS	AW_a	Index
1	-713.62	-0.041
2	-871.88	-0.047
3	**1,489.60**	**0.106**
4	705.32	0.044
5	2,368.42	0.095
6	-1,078.34	-0.028
7		
8		
9	-996.42	-0.046
10	-540.37	-0.040
11		
12		
13	-406.03	-0.031
14	-1,283.37	-0.112
15	-117.42	-0.007
16	-366.65	-0.035
17		
18		
19	-531.29	-0.037
20	-295.63	-0.016
21		
22	-183.31	-0.009
23	-7.48	0.000

CONSTRAINTS			
j	$\hat{Y}_j$	$\tilde{P}_j$	$\sum I_a$
1	0.988	0.0	
2	0.990	84.0	
3	0.980	83.2	
4	0.975	81.5	
5	0.965	112.0	
6	0.930	45.0	
7	0.920	41.9	
8	0.990	38.5	
9	0.988	105.1	
10	0.950	100.9	
11	0.954	95.8	
12	0.910	60.5	
		Total	**82,500**

SOLUTION SET = $OS(11,21)$
$OS(5,11)$
$OS(9,17)$
$OS(9,18)$
$OS(4,7)$

Figure 10. Continued.

$1,500, which is the remaining budget after the 7th strategy is picked, the heuristic solution has been achieved with an objective value of $15,173.54.

To estimate how close the heuristic solution is to the optimum value, UB_1 and UB_2 are calculated using Eq. (60) through (63):

$$UB_1 = 15{,}173.54 + \left\lfloor 1{,}500 \frac{1{,}489.60}{14{,}000} \right\rfloor = 15{,}332.54$$

with $s = OS(4,7)$ and $s+1 = OS(1,3)$

$$B_1 = 15{,}173.54 + \left\lfloor 1{,}500 \frac{2{,}368.42}{25{,}000} \right\rfloor = 15{,}315.54$$

with $s+2 = OS(3,5)$

$$B_2 = 1{,}489.60 + 13{,}333.32 = 14{,}822.92$$

with $s^* = OS(9,18)$

$$UB_2 = \max \{15{,}315.54\,;14{,}822.92\} = 15{,}315.54$$

As stated in theorem 2, UB_2 is smaller than UB_1. Moreover, the heuristic solution is at most 0.93% below the estimated optimum value UB_2. However, in this case, the heuristic solution is actually identical to the optimum solution presented in section 3.2.4 after applying the economic theory procedure.

The estimated results for the entire enterprise can be calculated using the last solution set on page 83. The total final profit R^* is equal to the total initial profit R plus the additional revenue minus the annual expenses for the selected operational strategies, or

$$R^* = R + \sum_{j=1}^{m} \left(EBC^*_j \, \Delta P_j\right) - \sum_{i=1}^{q} \left[AE_{a \mid OS(k,a)=1}\right] \qquad (64)$$

where EBC^*_j is the value of the EBCs under the optimum solution –those in the eighth iteration of the numerical example-, and ΔP_j is the quantity of material recovered at j^{th} process –values from the seventh iteration.- In this numerical example,

$$R^{*} = 153{,}440 = 128{,}209 + (102.903 \times 45 + 121.469 \times 42 + 133.762 \times 98 + 175.000 \times 36)$$
$$- (955 + 675 + 480 + 1{,}280 + 520)$$

The same result can be obtained from the EM model if the following two adjustments are made to account for the introduction of the new operational strategies: (1) the improved yield values $\hat{Y}_j$ are plugged in, and (2) the annual expenses are added to the fixed cost incurred per ton of product arriving at each one of the j^{th} processes affected by the new operational strategies –namely, 4, 5, 9 and 11- using the formula

$$\hat{Ft}_j = Ft_j + \frac{\sum_{a=1}^{q}\left[AE_{a \mid OS(k,a)=1 \,\&\, k=j}\right]}{\sum_{h=0}^{m} P_{h,j}} \tag{65}$$

where $\sum_{h=0}^{m} P_{h,j}$ represents the total product quantity arriving at the j^{th} process after the introduction of the new operational strategies. For instance,

$$Ft^{*}{}_{5} = 8.0 + \frac{675}{8{,}341} = 8.081$$

Table 11 shows the updated EM model, which also indicates that total energy consumption increased 6,904 kW·h/year (up 0.5% compared to a 19.7% rise in profit).

4.3.1.2 Numerical Example for Maintain-Throughput, Maximize-Profit Philosophy's Heuristic

Figure 11 shows three groups of information in worksheet format for each iteration of the heuristic in figure 9 when applied on the Type 1 hypothetical example in figure 6: *Operational Strategies* displays the EB, AW and index values for each operational strategy; *Constraints* presents the value of the constraint variables if the strategy with the highest index, which appears in bold style and over a gray background, is considered for inclusion in the solution; and *Solution Set* displays the strategies already selected.

Table 11. Type 1 EM Model's Numerical Example Under Optimum Solution for Increase-Throughput, Maximize-Profit Philosophy.

		TO PROCESS												Totals		TO FINAL PRODUCT					Totals
		1	2	3	4	5	6	7	8	9	10	11	12			1	2	3	4	5	
FROM PROCESS 0	*P*	10,000	0	0	0	0	0	0	0	0	0	0	0	10,000	*T*						
	r	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
	F	1	0	0	0	0	0	0	0	0	0	0	0								
1	*P*	0	9,800	0	0	0	0	0	0	0	0	0	0	9,800	*T*	0	0	0	0	0	0
	r	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
	F	0	1	0	0	0	0	0	0	0	0	0	0								
2	*P*	0	0	9,702	0	0	0	0	0	0	0	0	0	9,702	*T*	0	0	0	0	0	0
	r	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
	F	0	0	1	0	0	0	0	0	0	0	0	0								
3	*P*	0	0	0	9,508	0	0	0	0	0	0	0	0	9,508	*T*	0	0	0	0	0	0
	r	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
	F	0	0	0	1	0	0	0	0	0	0	0	0								
4	*P*	0	0	0	0	8,341	0	0	0	0	0	0	0	8,341	*T*	927	0	0	0	0	927
	r	0.00	0.00	0.00	0.00	0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.10	0.00	0.00	0.00	0.00	1.00
	F	0	0	0	0	1	0	0	0	0	0	0	0								
5	*P*	0	0	0	0	0	2,415	0	0	5,635	0	0	0	8,050	*T*	0	0	0	0	0	0
	r	0.00	0.00	0.00	0.00	0.00	0.30	0.00	0.00	0.70	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
	F	0	0	0	0	0	1	0	0	1	0	0	0								
6	*P*	0	0	0	0	0	0	2,246	0	0	0	0	0	2,246	*T*	0	0	0	0	0	0
	r	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
	F	0	0	0	0	0	0	1	0	0	0	0	0								
7	*P*	0	0	0	0	0	0	0	2,066	0	0	0	0	2,066	*T*	0	0	0	0	0	0
	r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
	F	0	0	0	0	0	0	0	1	0	0	0	0								
8	*P*	0	0	0	0	0	0	0	0	0	0	0	0	0	*T*	0	0	2,045	0	0	2,045
	r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	1.00	0.00	0.00	1.00
	F	0	0	0	0	0	0	0	0	0	0	0	0								
9	*P*	0	0	0	0	0	0	0	0	0	2,782	0	1,669	4,452	*T*	0	1,113	0	0	0	1,113
	r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.50	0.00	0.30		*r*	0.00	0.20	0.00	0.00	0.00	1.00
	F	0	0	0	0	0	0	0	0	0	1	0	1								
10	*P*	0	0	0	0	0	0	0	0	0	0	2,643	0	2,643	*T*	0	0	0	0	0	0
	r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
	F	0	0	0	0	0	0	0	0	0	0	1	0								
11	*P*	0	0	0	0	0	0	0	0	0	0	0	0	0	*T*	0	0	0	2,522	0	2,522
	r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	1.00	0.00	1.00
	F	0	0	0	0	0	0	0	0	0	0	0	0								
12	*P*	0	0	0	0	0	0	0	0	0	0	0	0	0	*T*	0	0	0	0	1,519	1,519
	r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	1.00	1.00
	F	0	0	0	0	0	0	0	0	0	0	0	0								
P Totals		10,000	9,800	9,702	9,508	8,341	2,415	2,246	2,066	5,635	2,782	2,643	1,669		*T*	927	1,113	2,045	2,522	1,519	8,126
Yield		**0.980**	**0.990**	**0.980**	**0.975**	**0.965**	**0.930**	**0.920**	**0.990**	**0.988**	**0.950**	**0.954**	**0.910**								
Energy/ton		**20**	**16**	**22**	**18**	**22**	**15**	**21**	**19**	**21**	**17**	**21**	**16**								
Energy		200,000	156,800	213,444	171,143	183,501	36,223	47,162	39,257	118,328	47,299	55,507	26,710	1,295,374							
Fixed cost/ton		**7.000**	**10.000**	**10.000**	**4.100**	**8.081**	**4.000**	**5.000**	**5.000**	**3.312**	**6.000**	**9.197**	**4.000**								
Variable cost/ton		**5.0**	**8.0**	**4.0**	**8.0**	**6.0**	**7.0**	**3.0**	**2.0**	**5.0**	**3.0**	**2.0**	**4.0**								
Accum. cost/ton		37.8	56.3	71.8	86.0	103.7	123.4	142.8	151.3	113.5	128.9	146.9	133.5								
Revenue/ton																**100.0**	**135.0**	**165.0**	**175.0**	**145.0**	
Profit																12,949	23,977	28,013	70,981	17,519	153,440

FIRST ITERATION

OPER. STRATEGIES			
OS	EB_j	AW_a	Index
1	2,027.60	-1,591.82	-0.091
2	1,915.54	-1,700.71	-0.092
3	3,144.48	113.94	0.008
4	3,080.05	-204.14	-0.013
5	6,115.48	889.87	0.036
6	5,268.74	-2,343.99	-0.061
7	3,866.60	1,076.30	0.086
8	8,541.75	2,774.32	0.107
9	3,267.59	-1,639.13	-0.076
10	1,980.97	-926.16	-0.069
11	4,367.01	2,150.36	0.205
12	4,039.32	1,974.25	0.179
13	2,584.22	-674.50	-0.052
14	1,482.64	-1,430.84	-0.124
15	2,846.51	-397.91	-0.023
16	2,266.80	-574.85	-0.055
17	4,034.07	1,792.18	0.149
18	7,224.18	2,787.46	0.130
19	1,619.87	-784.08	-0.054
20	2,611.36	-803.30	-0.042
21	**5,380.48**	**3,098.59**	**0.258**
22	3,306.96	-726.35	-0.035
23	2,688.97	-218.51	-0.014

CONSTRAINTS		
j	$\hat{Y}_j$	$\sum I_a$
1	0.980	
2	0.990	
3	0.980	
4	0.970	
5	0.960	
6	0.930	
7	0.920	
8	0.990	
9	0.970	
10	0.950	
11	0.954	
12	0.910	
	Total	12,000

SOLUTION SET = *OS*(11,21)

SECOND ITERATION

OPER. STRATEGIES			
OS	EB_j	AW_a	Index
1	2,018.23	-1,601.19	-0.091
2	1,906.69	-1,709.56	-0.092
3	3,129.94	99.40	0.007
4	3,065.80	-218.39	-0.014
5	6,087.19	861.58	0.034
6	5,244.38	-2,368.35	-0.062
7	3,848.72	1,058.42	0.085
8	8,502.25	2,734.82	0.105
9	3,252.49	-1,654.23	-0.077
10	1,971.81	-935.32	-0.069
11	**4,344.72**	**2,128.07**	**0.203**
12	4,039.54	1,974.47	0.179
13	2,584.36	-674.36	-0.052
14	1,482.73	-1,430.75	-0.124
15	2,846.68	-397.74	-0.023
16	2,266.94	-574.71	-0.055
17	4,004.55	1,762.66	0.147
18	7,171.31	2,734.59	0.127
19	1,608.02	-795.93	-0.055
20	2,573.08	-841.58	-0.044
21			
22	3,211.06	-822.25	-0.039
23	2,689.03	-218.45	-0.014

CONSTRAINTS		
j	$\hat{Y}_j$	$\sum I_a$
1	0.980	
2	0.990	
3	0.980	
4	0.970	
5	0.965	
6	0.930	
7	0.920	
8	0.990	
9	0.970	
10	0.950	
11	0.954	
12	0.910	
	Total	22,500

SOLUTION SET = *OS*(11,21)
OS(5,11)

Figure 11. Philosophy 2 Non-Relaxed Heuristic's Numerical Example Worksheets.

THIRD ITERATION

OPER. STRATEGIES			
OS	EB_j	AW_a	Index
1	2,008.71	-1,610.71	-0.092
2	1,897.69	-1,718.56	-0.093
3	3,115.17	84.63	0.006
4	3,051.34	-232.85	-0.015
5	6,058.48	832.87	0.033
6	5,219.64	-2,393.09	-0.062
7	3,830.56	1,040.26	0.083
8	8,462.14	2,694.71	0.104
9	3,237.14	-1,669.58	-0.078
10	1,962.50	-944.63	-0.070
11			
12	**4,020.38**	**1,955.31**	**0.178**
13	2,572.10	-686.62	-0.053
14	1,475.69	-1,437.79	-0.125
15	2,833.99	-410.43	-0.023
16	2,257.30	-584.35	-0.056
17	3,985.04	1,743.15	0.145
18	7,136.38	2,699.66	0.126
19	1,600.19	-803.76	-0.055
20	2,561.45	-853.21	-0.045
21			
22	3,197.66	-835.65	-0.040
23	2,676.78	-230.70	-0.015

CONSTRAINTS		
j	$\hat{Y}_j$	$\sum I_a$
1	0.980	
2	0.990	
3	0.980	
4	0.970	
5	0.965	
6	0.944	
7	0.920	
8	0.990	
9	0.970	
10	0.950	
11	0.954	
12	0.910	
	Total	33,500

SOLUTION SET = $OS(11,21)$
$OS(5,11)$
$OS(6,12)$

FOURTH ITERATION

OPER. STRATEGIES			
OS	EB_j	AW_a	Index
1	2,000.74	-1,618.68	-0.092
2	1,890.16	-1,726.09	-0.093
3	3,102.81	72.27	0.005
4	3,039.23	-244.96	-0.015
5	6,034.45	808.84	0.032
6	5,198.93	-2,413.80	-0.063
7	3,815.37	1,025.07	0.082
8	8,428.58	2,661.15	0.102
9	3,244.30	-1,662.42	-0.077
10	1,954.71	-952.42	-0.071
11			
12			
13	2,497.75	-760.97	-0.059
14	1,432.94	-1,480.54	-0.129
15	2,795.05	-449.37	-0.026
16	2,227.73	-613.92	-0.058
17	**3,985.04**	**1,743.15**	**0.145**
18	7,136.38	2,699.66	0.126
19	1,600.18	-803.77	-0.055
20	2,561.44	-853.22	-0.045
21			
22	3,197.66	-835.65	-0.040
23	2,676.77	-230.71	-0.015

CONSTRAINTS		
j	$\hat{Y}_j$	$\sum I_a$
1	0.980	
2	0.990	
3	0.980	
4	0.970	
5	0.965	
6	0.944	
7	0.920	
8	0.990	
9	0.976	
10	0.950	
11	0.954	
12	0.910	
	Total	45,500

SOLUTION SET = $OS(11,21)$
$OS(5,11)$
$OS(6,12)$
$OS(9,17)$

Figure 11. Continued.

FIFTH ITERATION

OPER. STRATEGIES			
OS	EB_j	AW_a	Index
1	1,992.63	-1,626.79	-0.093
2	1,882.50	-1,733.75	-0.094
3	3,090.24	59.70	0.004
4	3,026.92	-257.27	-0.016
5	6,010.00	784.39	0.031
6	5,177.87	-2,434.86	-0.063
7	3,799.91	1,009.61	0.081
8	8,394.43	2,627.00	0.101
9	3,211.24	-1,695.48	-0.079
10	1,946.79	-960.34	-0.071
11			
12			
13	2,497.75	-760.97	-0.059
14	1,432.95	-1,480.53	-0.129
15	2,795.06	-449.36	-0.026
16	2,227.74	-613.91	-0.058
17			
18	**7,045.46**	**2,608.74**	**0.121**
19	1,579.71	-824.24	-0.057
20	2,546.17	-868.49	-0.046
21			
22	3,180.07	-853.24	-0.041
23	2,660.69	-246.79	-0.016

CONSTRAINTS		
j	$\hat{Y}_j$	$\sum I_a$
1	0.980	
2	0.990	
3	0.980	
4	0.970	
5	0.965	
6	0.944	
7	0.920	
8	0.990	
9	0.976	
10	0.950	
11	0.954	
12	0.910	
	Total	67,000

SOLUTION SET = $OS(11,21)$
$OS(5,11)$
$OS(6,12)$
$OS(9,17)$
$OS(9,18)$

SIXTH ITERATION

OPER. STRATEGIES			
OS	EB_j	AW_a	Index
1	1,978.30	-1,641.12	-0.094
2	1,868.96	-1,747.29	-0.094
3	3,068.01	37.47	0.003
4	3,005.15	-279.04	-0.017
5	5,966.77	741.16	0.030
6	5,140.62	-2,472.11	-0.064
7	3,772.58	982.28	0.079
8	**8,334.05**	**2,566.62**	**0.099**
9	3,188.14	-1,718.58	-0.080
10	1,932.79	-974.34	-0.072
11			
12			
13	2,497.75	-760.97	-0.059
14	1,432.95	-1,480.53	-0.129
15	2,795.06	-449.36	-0.026
16	2,227.74	-613.91	-0.058
17			
18			
19	1,543.83	-860.12	-0.059
20	2,519.17	-895.49	-0.047
21			
22	3,148.97	-884.34	-0.042
23	2,632.24	-275.24	-0.017

CONSTRAINTS		
j	$\hat{Y}_j$	$\sum I_a$
1	0.980	
2	0.990	
3	0.980	
4	0.970	
5	0.965	
6	0.944	
7	0.920	
8	0.990	
9	0.976	
10	0.950	
11	0.954	
12	0.910	
	Total	93,000

SOLUTION SET = $OS(11,21)$
$OS(5,11)$
$OS(6,12)$
$OS(9,17)$
$OS(9,18)$

Figure 11. Continued.

The first iteration, whose EB values are identical to those in table 9, selects the 21st strategy. Then, the second through fifth iterations select the 11th, 12th, 17th and 18th strategies. Finally, the sixth iteration attempts to select the 8th strategy, but this strategy violates the budget constraint. In fact, no other strategy can be selected since only $3,000 of budget is available at this point. The heuristic solution has been achieved with an objective value of $11,060.57. It should be noted that this heuristic solution is also identical to the optimal solution presented in section 3.3.4 after utilizing the economic theory procedure.

To obtain the same result from the EM model, a number of adjustments have to be made as follows (values with gray background in table 12):

a) Reduce total material input.
b) Modify ratio values $\hat{r}_{j,k,i}$ to maintain total material output for each final product.
c) Increase yield values $\hat{Y}_j$ according to the selected strategies' expected annual expenses.
d) Adjust fixed cost values $\hat{Ft}_j$.

Table 12 displays the updated EM model, which indicates that total raw material input decreases 111 tons and energy consumption reduces 12,973 kW·h/year (down 1.0% compared to a 16.7% increase in profit).

4.3.2 Multi-Dimensional Zero-One Knapsack Heuristic Method

A detailed description of the Lau's algorithm (1986), which is particularly effective for solving large size problems, is presented next for the problem

$$\text{Maximize} \quad \sum_{a=1}^{q}\left(AW_a\, OS(j,a)\right)$$

$$\text{Subject to} \quad \sum_{a=1}^{q}\left(\Delta P_{j(a)}\, OS(j,a)\right) \le (1-Y_j)\sum_{k=0}^{m} P_{k,j} = b_j \qquad \text{for } j=1,2,\ldots,m$$

$$\sum_{a=1}^{q}\left(I_a\, OS(j,a)\right) \le B = b_{m+1}$$

$$OS(j,a) \in \{0,1\} \qquad \text{for } a=1,2,\ldots,q$$

Table 12. Type 1 EM Model's Numerical Example Under Optimum Solution for Maintain-Throughput, Maximize-Profit Philosophy.

			TO PROCESS												Totals		TO FINAL PRODUCT					Totals
			1	2	3	4	5	6	7	8	9	10	11	12			1	2	3	4	5	
FROM PROCESS	0	*P*	9,889	0	0	0	0	0	0	0	0	0	0	0	9,889	*T*						
		r	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	1	0	0	0	0	0	0	0	0	0	0	0								
	1	*P*	0	9,691	0	0	0	0	0	0	0	0	0	0	9,691	*T*	0	0	0	0	0	0
		r	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	0	1	0	0	0	0	0	0	0	0	0	0								
	2	*P*	0	0	9,594	0	0	0	0	0	0	0	0	0	9,594	*T*	0	0	0	0	0	0
		r	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	0	0	1	0	0	0	0	0	0	0	0	0								
	3	*P*	0	0	0	9,402	0	0	0	0	0	0	0	0	9,402	*T*	0	0	0	0	0	0
		r	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	0	0	0	1	0	0	0	0	0	0	0	0								
	4	*P*	0	0	0	0	8,198	0	0	0	0	0	0	0	8,198	*T*	922	0	0	0	0	922
		r	0.00	0.00	0.00	0.00	0.899	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.101	0.00	0.00	0.00	0.00	1.00
		F	0	0	0	0	1	0	0	0	0	0	0	0								
	5	*P*	0	0	0	0	0	2,356	0	0	5,556	0	0	0	7,911	*T*	0	0	0	0	0	0
		r	0.00	0.00	0.00	0.00	0.00	0.298	0.00	0.00	0.702	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	0	0	0	0	0	1	0	0	1	0	0	0								
	6	*P*	0	0	0	0	0	0	2,223	0	0	0	0	0	2,223	*T*	0	0	0	0	0	0
		r	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	0	0	0	0	0	0	1	0	0	0	0	0								
	7	*P*	0	0	0	0	0	0	0	2,045	0	0	0	0	2,045	*T*	0	0	0	0	0	0
		r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	0	0	0	0	0	0	0	1	0	0	0	0								
	8	*P*	0	0	0	0	0	0	0	0	0	0	0	0	0	*T*	0	0	2,025	0	0	2,025
		r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	1.00	0.00	0.00	1.00
		F	0	0	0	0	0	0	0	0	0	0	0	0								
	9	*P*	0	0	0	0	0	0	0	0	0	2,666	0	1,623	4,289	*T*	0	1,082	0	0	0	1,082
		r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.486	0.00	0.296		*r*	0.00	0.197	0.00	0.00	0.00	0.98
		F	0	0	0	0	0	0	0	0	0	1	0	1								
	10	*P*	0	0	0	0	0	0	0	0	0	0	2,532	0	2,532	*T*	0	0	0	0	0	0
		r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00		*r*	0.00	0.00	0.00	0.00	0.00	1.00
		F	0	0	0	0	0	0	0	0	0	0	1	0								
	11	*P*	0	0	0	0	0	0	0	0	0	0	0	0	0	*T*	0	0	0	2,416	0	2,416
		r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	1.00	0.00	1.00
		F	0	0	0	0	0	0	0	0	0	0	0	0								
	12	*P*	0	0	0	0	0	0	0	0	0	0	0	0	0	*T*	0	0	0	0	1,477	1,477
		r	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00		*r*	0.00	0.00	0.00	0.00	1.00	1.00
		F	0	0	0	0	0	0	0	0	0	0	0	0								
P Totals			9,889	9,691	9,594	9,402	8,198	2,356	2,223	2,045	5,556	2,666	2,532	1,623		*T*	922	1,082	2,025	2,416	1,477	7,922
Yield			0.980	0.990	0.980	0.970	0.965	0.944	0.920	0.990	0.988	0.950	0.954	0.910								
Energy/ton			20	16	22	18	22	15	21	19	21	17	21	16								
Energy			197,770	155,052	211,064	169,235	180,353	35,336	46,690	38,864	116,670	45,315	53,178	25,971	1,275,497							
Fixed cost/ton			7.000	10.000	10.000	4.000	8.082	4.191	5.000	5.000	3.317	6.000	9.205	4.000								
Variable cost/ton			5.0	8.0	4.0	8.0	6.0	7.0	3.0	2.0	5.0	3.0	2.0	4.0								
Accum. cost/ton			37.8	56.3	71.8	86.3	104.1	122.1	141.4	149.9	113.8	129.3	147.2	133.8								
Revenue/ton																	100.0	135.0	165.0	175.0	145.0	
Profit																	12,591	22,949	30,518	67,082	16,486	149,626

Let

J = set of all q strategies,

S = set of strategies already included in the solution,

T = set of strategies not in S, i.e., $T = J - S$, and

R = $(m+1)$-vector of the cumulative total resource vector required by S.

Step 1 (Initialize)

Compute the ratios

$$d_{j,a} = \Delta P_{j(a)} / b_j \qquad \text{for } j=1,2,\ldots,m \text{ and } a=1,2,\ldots,q$$

$$d_{m+1,a} = I_a / b_{m+1} \qquad \text{for } a=1,2,\ldots,q$$

Let D_a be the resource requirement vector for strategy a,

$$D_a = (d_{1,a}; d_{2,a}; \ldots; d_{m,a}; d_{m+1,a})$$

and N be the normalized $(m+1)$ limit vector of resources,

$$N = (1, 1, \ldots, 1)$$

Initialize

$$S = \vec{0},$$

$$T = J,$$

$$R = \vec{0},$$

$$Z = 0, \text{ and}$$

$$OS(j,a) = 0 \qquad \text{for } a=1,2,\ldots,q.$$

Step 2 (Stopping criterion)

Let U be the set of all eligible strategies,

$$U = \{OS(j,a) : OS(j,a) \in T \text{ and } D_a \le N - R\}$$

If U is empty, then stop.

Step 3 (Calculate gradients)

If $R = \vec{0}$, compute the effective gradients g_a for strategies in U as

$$g_a = \frac{AW_a \sqrt{m+1}}{\sum_{j=1}^{m+1} d_{j,a}} \qquad \text{for } a \in U$$

Otherwise, compute

$$f_j = \sum_{a \in U} d_{j,a} \qquad \text{for } j=1,2,\ldots,m+1$$

$$g_a = \frac{AW_a \sqrt{\sum_{j=1}^{m+1} f_j^2}}{\sum_{j=1}^{m+1} (d_{j,a} f_j)} \qquad \text{for } a \in U$$

In the case when

$$\sum_{j=1}^{m+1} d_{j,a} f_j = 0$$

then set

$$g_a = +\infty$$

<u>Step 4 (Add strategy to solution)</u>

Among the effective gradients, let g_k be the largest, and set

$$S = S + \{k\},$$

$$T = T - \{k\},$$

$$R = R + D_k,$$

$$OS(j,k) = 1.$$

Recalculate AW_a values and compute

$$Z = \sum_{a \in S} AW_a$$

Go to step 2.

The algorithm calculates a *gradient* in step 3, which is equivalent to the contribution that each strategy provides to the objective function divided by weighted average consumption of all resources, and selects the available operational strategy with the highest gradient. The objective's function contribution is obviously proportional to AW_a; while the weighted average consumption of resources, where b_j represents the upper limit on j^{th} resource, is proportional to $\sum_{j=1}^{m+1} d_{j,a}$.

4.3.3 Zero-One Knapsack Branch and Bound Method

The first branch-and-bound approach to the exact solution of the zero-one Knapsack problem was proposed by Kolesar (1967). Since then, two main streams of zero-one Knapsack branch-and-bound algorithms have emerged depending upon the number of objects in the problem. If this number is smaller or equal to 200, algorithms that first sort objects are preferred. However, due to the fact that when the number of objects is larger than 2,000 about 80% of the total time required by those algorithms represents sorting time, a second set of algorithms that do not require preliminary sorting has been devised to be applied when the number of objects is larger than 500 (Martello and Toth, 1990). Since the number of new operational strategies to be considered by a steel manufacturer is not expected to exceed 200 for a single run of the EMP models, the former group of algorithms will be considered in this research.

Two main streams of algorithms that overcome the large computer memory and time requirements of Kolesar's algorithm have been proposed (Christofides et al, 1979; Martello and Toth, 1990). The first one started with Greenberg and Hegerich (1970), who used a depth-first type of strategy instead of the breadth-first binary branching scheme adopted by Kolesar. Horowitz and Sahni (1974) introduced the second one by also using a depth-first scheme but keeping the branching variable selection process used by Kolesar. The latter, however, is more efficient, structured and easy to implement. In addition, several algorithms have been obtained by improving the Horowitz-Sahni strategy, but the algorithm proposed by Martello and Toth (1977) is generally considered the most effective.

A detailed description of the Martello-Toth algorithm is included below for the problem

$$\text{Maximize} \quad \sum_{a=1}^{q}\left(AW_a\, OS(j,a)\right)$$

$$\text{Subject to} \quad \sum_{a=1}^{q}\left(I_a\, OS(j,a)\right) \leq B$$

$$OS(j,a) \in \{0,1\} \qquad \text{for } a=1,2,\ldots,q$$

Step 1 (Initialize)

1.1 Order the strategies in decreasing order of AW_a/I_a.

1.2 Compute $AW^* = \sum_{a=1}^{S} AW_a$ with S = largest index for which $I^* = \sum_{a=1}^{S} I_a \leq B$.

1.3 If $I^* = B$, the optimal solution is given by $P = AW^*$, with $X_a = 1$ for $a = 1,\ldots,S$ and $X_a = 0$ for a = S+1,…,q. Stop.

1.4 If $I^* < B$, compute $M_a = \min\{I_k \mid a < k \leq q\}$ for $a = 1,\ldots,q$-1 and $M_q =$. Set $U = UB_2$ (see Eq. (4)), $p = P = 0$ with $x_a = 0$ for $a = 1,\ldots,q$, $i = 1$, $\overline{S} = q$. Go to step 4.

Step 2 (Try to insert the i^{th} strategy into the current solution)

2.1 If I_i B, go to step 3.

2.2 If P $p + \left\lfloor B\left(AW_{i+1}/I_{i+1}\right)\right\rfloor$, go to step 5. Otherwise, set $i = i + 1$ and repeat step 2.

Step 3 (Build a new current solution)

3.1 Compute $AW^* = \overline{p}_i + \sum_{a=\overline{z}_i}^{S} AW_a$ with S = largest index for which $I^* = \overline{w}_i + \sum_{a=z_i}^{S} I_a \leq B$ and $S \leq q$. If $\overline{w}_i + I_{\overline{z}_i} > B$, set $S = \overline{z}_i - 1$. Two possibilities exist:

a) $I^* < B$ and $S < q$: if $P \geq p + AW^* + \left\lfloor (B - I^*) AW_{S+1}/I_{S+1} \right\rfloor$, go to step 6. Otherwise, go to step 4.

b) $I^* = B$ or $S = q$: if $P \geq p + AW^*$, go to step 6. Otherwise, set $P = p + AW^*$ with $X_a = x_a$ for $a = 1,\ldots,i-1$, $X_a = 1$ for $a = i,\ldots,S$, and $X_a = 0$ for $a = S+1,\ldots,q$. If $P = U$, stop. Otherwise, go to step 6.

Step 4 (Save the current solution)

4.1 Set $B = B - I^*$, $p = p + AW^*$ with $x_a = 1$ for $a = i,\ldots,S$.

4.2 Compute $\overline{w}_i = I^*$, $\overline{p}_i = AW^*$, $\overline{z}_i = S + 1$; with $\overline{w}_a = \overline{w}_{a-1} - I_{a-1}$, $\overline{p}_a = \overline{p}_{a-1} - AW_{a-1}$, and $\overline{z}_a = S + 1$ for $a = i+1,\ldots,S$; $\overline{w}_a = \overline{p}_a = 0$ and $\overline{z}_a = a$ for $a = S+1,\ldots,\overline{S}$. Three possibilities exist:

a) $S < q - 2$: set $i = S + 2$. If $B < M_{i-1}$, go to step 5. Otherwise, go to step 2.

b) $S = q-2$: if $B \geq I_q$, set $B = B - I_q$, $p = p + AW_q$, $x_q = 1$. In any case, set $i = q-1$ and go to step 5.

c) $S = q-1$: set $i = q$ and go to step 5.

Step 5 (Save the current optimal solution)

5.1 If $P < p$, set $P = p$ with $X_a = x_a$ for $a = 1,\ldots,q$.

5.2 If $P = U$, stop.

5.3 If $P \geq p$ or $P \neq U$, and if $x_q = 1$; set $B = B - w_q$, $p = p - AW_q$, $x_q = 0$.

5.4 Go to step 6.

Step 6 (Backtrack)

6.1 Find the largest $k < i$ for which $x_k = 1$. If no such k exists, stop.

6.2 Set $R = B$, $B = B + I_k$, $p = p - AW_k$, $x_k = 0$.

6.3 If $R \geq M_k$, set $i = k+1$ and go to step 2. Otherwise, set $i = k$, $h = k+1$ and go to step 7.

Step 7 (Try to substitute the h^{th} strategy for the k^{th})

7.1 If $h > q$ or $P \geq p + \left\lfloor B\left(AW_h / I_h\right)\right\rfloor$, go to step 6.

7.2 Set $D = I_h - I_k$. Three possibilities exist:

a) $D = 0$: set $h = h+1$ and repeat step 7.

b) $D > 0$: if $D > R$ or $P \geq p + AW_h$, set $h = h+1$ and repeat step 7. Otherwise, set

$P = p + AW_h$ with $X_a = x_a$ for $a = 1,\ldots,k$, $X_a = 0$ for $a = k+1,\ldots,q$ ($j \neq h$), $X_h = 1$.

If $P = U$, stop. Otherwise, set $R = R - D$, $k = h$, $h = h+1$ and repeat step 7.

c) $D < 0$: if $R - D < M_h$, set $h = h+1$ and repeat step 7. Otherwise, if

$P \geq p + AW_h + \left\lfloor (R-D)\, AW_h / I_h \right\rfloor$, go to step 6. Otherwise, set $B = B + I_h$,

$p = p + AW_h$, $x_h = 1$, $i = h+1$, $\overline{w}_h = I_h$, $\overline{p}_h = AW_h$, $\overline{z}_h = h+1$, $\overline{S} = h$; and

$\overline{w}_a = \overline{p}_a = 0$, $\overline{z}_a = a$ for $a = h+1,\ldots,\overline{S}$ and go to step 2.

In this algorithm, a *forward move* –steps 2, 3 and 4- consists of inserting the largest set of new consecutive strategies into the current solution. In step 3, where the new current solution is built, case (b) saves a new optimal solution if it is worthwhile but it does not update the vector x_j to avoid needless backtracking on values $x_i,\ldots x_s$. On the other hand, case (a) performs step 4 if the current solution found can improve on the current optimal solution through subsequent forward moves; otherwise, a backtracking step follows.

A *backtracking move* consists of removing the k^{th} strategy, which is followed by a normal forward move only if R (value of B preceding the backtracking) is large enough to allow the introduction into the solution of at least one of the strategies following the k^{th}. Otherwise, the particular forward procedure in step 7 is utilized.

4.3.4 Local Search Method to Improve Solution

Local or neighborhood search consists of moving from a solution to another one in its neighborhood according to some well-defined rules (Pirlot, 1996). In other words, a local search strategy starts from an arbitrary or heuristic solution $x_1 \in X$ and, at each step n, a new solution x_{n+1} is chosen in the neighborhood $N(x_n)$ of the current solution x_n, where a neighborhood $N(x)$ on X is defined as a subset $N(x) \subseteq X$ for each $x \in X$. For example, if X is a set of binary vectors and $x \in X$, $N(x)$ can be defined as the set of all solutions $x \in X$ obtained by flipping a single coordinate from 0 to 1 or conversely. Conventionally, it is assumed that a solution does not belong to its own neighborhood, i.e., $x \notin N(x)$, $\forall\, x \in X$. The steps of the generic local search algorithm for a maximization problem are described below (Reeves, 1993):

Step 1 (Initialization)

Select a starting solution $x^{now} \in X$, record the current best solution by setting $x^{best} = x^{now}$, and define $f^{best} = f(x^{now})$.

Step 2 (Choice and termination)

Choose a solution $x^{next} \in N(x^{now})$ according to some choice criteria. If the choice criteria employed cannot be satisfied by any member of $N(x^{now})$, or if any other termination criteria apply, then the algorithm stops.

Step 3 (Update)

Set $x^{now} = x^{next}$; and if $f^{best} < f(x^{now})$, set $f^{best} = f(x^{now})$.

Obviously, the criteria chosen for selecting moves and for terminating the algorithm in the generic search method yield a variety of procedures. For instance, *descent* methods only permit moves to neighbor solutions that improve the current f^{best} value and end when no improving solutions can be found. Likewise, *Monte Carlo* methods, which include simulated annealing, normally select the next move through an exponential function to define probabilities.

Two main aims of local search methods are (1) to find the global optimum by evading local optima; and (2) to avoid cycling, a phenomenon that makes a local search strategy oscillate between solutions without improving the objective function as explained next. Even if there is no better solution than the current one, x_n, in the neighborhood $V(x_n)$, the search method moves to the best possible solution x in $V(x)$. If the neighborhood structure is symmetric, i.e., if x_n belongs to the neighborhood $V(x)$ of x whenever $x \in V(x_n)$, there is a chance x_n could be the best solution in $V(x)$ in which case the search method would come back to x_n, and from then on, oscillate between x and x_n. In some cases, cycling can occur after a number of moves.

Tabu search, which was created by Glover (1986) and independently by Hansen (1986), has proven to be an extremely efficient and flexible local search method to escape local optima and to avoid cycling (Glover, Taillard and de Werra, 1993).

The philosophy of Tabu search is to derive and exploit a collection of principles of intelligent problem solving. As a result, the fundamental element underlying Tabu search is the use of certain forms of flexible memory that impose restrictions to guide a search process to negotiate otherwise difficult regions in the search area. A chief mechanism for exploiting flexible memory in Tabu search is to classify a subset of the moves in the neighborhood as forbidden or tabu. This classification depends on the history of the search, particularly as manifested in the recency and/or frequency that certain moves have participated in generating past solutions (Reeves, 1993).

Recency basically prevents moves that have been performed in the recent past to take place again until their tenure, number of iterations after their last appearance, has been completed. Frequency, on the other hand, allows us to diversify the search by driving it into new regions. This diversifying influence is usually restricted to operate only on particular occasions – for instance, when there are no admissible improving moves, it favors non-improving moves with lower frequency counts. Furthermore, Tabu search allows an important exception called the *aspiration* *criterion* to determine when tabu restrictions can be overridden. For example, when a tabu move would result in a solution better than any visited so far, its tabu classification may be overridden.

The effect of flexible memory may be envisioned by stipulating that Tabu search maintains a selective history H of the states encountered during the search, and replaces $N(x^{now})$ with a modified neighborhood that may be denoted as $N(H,x^{now})$. History then determines which solutions may be reached by a move from the current solution, selecting x^{next} from $N(H,x^{now})$. If the Tabu search strategy is based on short term considerations, $N(H,x^{now})$ is typically a subset of $N(x^{now})$. In the intermediate and long term strategies, $N(H,x^{now})$ may contain solutions not in $N(x^{now})$.

The generic Tabu search method can be expressed in the following manner:

Step 1 (Initialization)

Select a starting solution $x^{now} \in X$, record the current best solution by setting $x^{best} = x^{now}$, and define $f^{best} = f(x^{now})$ and H as empty.

Step 2 (Choice and termination)

Choose a solution $x^{next} \in N(H,x^{now})$ such that $f(x^{next}) - f(x^{now}) > 0$ and $[f(x^{next}) - f(x^{now})]$ is greatest. If there is not a $x^{next} \in N(H,x^{now})$ such that $f(x^{next}) - f(x^{now}) > 0$, diversify the search by choosing a $x^{next} \in N(H,x^{now})$ such that $[f(x^{now}) - f(x^{next})]$ is smallest. Terminate by a chosen iteration cut-off rule.

Step 3 (Update)

Set $x^{now} = x^{next}$; and if $f^{best} < f(x^{now})$, set $f^{best} = f(x^{now})$. Update H.

A specific Tabu search method, such as the one used in this research, depends on how $N(x^{now})$ is obtained, on how the history record H is defined and used, on how the neighborhood $N(H,x^{now})$ is determined, and on the stopping criteria as discussed below.

In this research, $N(x^{now})$ is obtained by switching, one by one, each operational strategy $OS(j,a)$ in the current solution from one to zero, and by rebuilding all possible feasible solutions through the addition of operational strategies not in the current solution. In order to minimize computational time, feasible solutions are rebuilt using the search enumeration technique explained in section 4.3.5.

The history record H can be defined as the collection of the $\lceil 2\sqrt{q} \rceil$ solutions most recently visited, where q is the total number of new operational strategies being considered and $\lceil z \rceil$ is the smallest integer greater than or equal to z. This recency-based memory will prevent cycles of length less than or equal to $\lceil 2\sqrt{q} \rceil$ from occurring in the trajectory followed in moving from one solution to the next. In addition, this tabu list yields

$$N(H,x^{now}) = N(x^{now}) - H.$$

However, this *explicit* memory approach, where complete solutions are stored, generally consumes a significant amount of computer memory and time.

An alternative is to use *attribute-based recency* memory. An attribute of a move from x^{now} to x^{next} encompasses any aspect that changes as a result of the move. Two natural types of attributes are:

(a) the change of a variable $OS(j,a)$ from 0 to 1, and
(b) the change of a variable $OS(j,a)$ from 1 to 0.

These move attributes are then used to impose two tabu restrictions:

(a) $OS(j,a)$ changes from 1 to 0 (where $OS(j,a)$ previously changed from 0 to 1), and
(b) $OS(j,a)$ changes from 0 to 1 (where $OS(j,a)$ previously changed from 1 to 0).

To keep track of the status of move attributes that compose the tabu restrictions, and to determine when these restrictions are applicable; a recency-based function, specified by the array

$$tabuend(a) = Iter + tabutenure + 1$$

if $OS(j,a)$ was dropped (changed from 1 to 0) or added (changed from 0 to 1) during iteration *Iter* and must remain tabu-active for a tabu tenure (number of iterations) *tabutenure*, may be used. Thus, *tabuend* records the iteration where a strategy $OS(j,a)$ ends its tabu-active status. A key issue for creating tabu status using recency-based memory is to determine a "good" value for *tabutenure*. Tabu tenure values between 7 and 20, as well as equal to $\lceil \sqrt{q} \rceil$, appear to work well for a variety of problems (Glover and Laguna, 1997).

The alternative memory approach presented above yields

$$N(H,x^{now}) = \{ x^{next} \mid \exists\ m \in M(x^{now}) \text{ with } x^{next} = x^{now} \oplus m \}$$

where H is the array *tabuend*(a) for a=1,...,q and m is a legal (non-tabu) move in the set $M(x^{now})$ as specified by the active tabu restrictions included in H.

Finally, the stopping criteria will be twofold:

(a) $N(H,x^{now})$ is empty, or
(b) the number of iterations performed since f^{best} last changed is greater than 5.

4.3.5 Local Search Method to Find Feasible Solution

If the solution of the relaxed problem is infeasible for the original problem –the one with capacity constraints-, it is necessary to find a feasible solution. A two-step local search method based is proposed next to accomplish this.

The first step is built around two concepts. First, since the original problem and the relaxed problem share the same linear objective function, a feasible solution for the original problem is expected to be located nearby the unfeasible solution found through the relaxed problem. Second, if the relaxed-problem solution exceeds one or more capacity constraints, one or more of the basic decision variables –a decision variable whose value is equal to one- in the unfeasible solution is causing it. Consequently, the local search's first step starts by making basic decision variables non-basic (equal to zero) one at a time and checking whether the resulting modified solution is feasible. If none of these modified solutions is feasible, the method then makes pairs of basic decision variables non-basic and so on until at least one feasible solution is discovered.

The local search's second step then checks whether each modified feasible solution may be improved by fixing the values of the original basic decision variables in the modified solution and generating improved solutions through the inclusion of non-basic decision variables.

Summing up, the first step is a *back step* to return to feasibility, and the second step is a *forward step* to improve the feasible solution as presented in figure 12. To numerically illustrate this local search method, the solution for the following problem after relaxing the first constraint is (0,1,1,0,1) with an objective value of 25,

$$\begin{aligned}
\text{Max} \quad & 2x_1 + 12x_2 + 5x_3 + 4x_4 + 8x_5 \\
\text{Subject to} \quad & 2x_1 + 4x_2 + 2x_3 + 3x_4 + 4x_5 \leq 9 \\
& x_1 + x_2 + x_3 + x_4 + x_5 \leq 3 \\
& x_1, x_2, x_3, x_4, x_5 = 0,1
\end{aligned}$$

However, this solution is unfeasible for the original problem since it violates the first constraint. So the basic variable x_2 is set equal to zero (back step) to generate the modified feasible solution

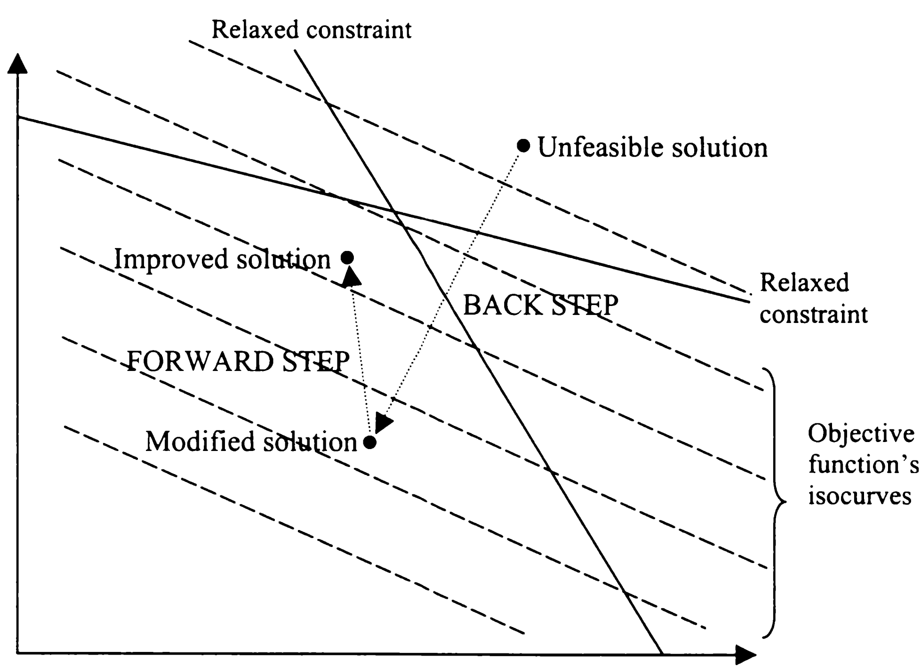


Figure 12. Local Search Method to Find Feasible Solution.

(0,0,1,0,1) with a lower objective function value of 13. Then, with the values of variables x_2, x_3 and x_5 fixed; three combinations of non-basic variables x_1 and x_4 can be generated (forward step) in an attempt to create three improved solutions – see rows 2 through 5 in the table 13. Obviously, the second improved solution (0,0,1,1,1) provides the best objective function value so far. After the local search method is also applied to basic variables x_3 and x_5 –rows 6 through 13 in table 13,- the best feasible solution found is (0,1,1,1,0) with an objective function value of 21.

In order to reduce the number of solutions to be considered during the forward step, the following search enumeration technique will be used (Winston, 1987) - a search enumeration technique creates a *search tree* and exploits the fact that the decision variables must equal zero or one to efficiently determine whether further branching at a node is required thus avoiding an explicit enumeration.

The technique starts building the search tree by branching on a free *OS(j,a)* decision variable and adding two nodes: a node with *OS(j,a)* equal to zero and a node with *OS(j,a)* equal to one - a *free* decision variable is a decision variable whose value is still unspecified, while a *fixed* decision variable is a decision variable whose value has been specified. Consequently, as the tree grows, each branch will specify for some decision variables *OS(j,a)* whether they are equal to zero or

Table 13. Numerical Example for the Local Search Method to Find Feasible Solution.

x_1	x_2	x_3	x_4	x_5	Solution feasible?	Objective function
0	1	1	0	1	No	25
0	0	1	0	1	Yes	13
1	0	1	0	1	Yes	15
0	0	1	1	1	Yes	17
1	0	1	1	1	No	
0	1	0	0	1	Yes	20
1	1	0	0	1	No	
0	1	0	1	1	No	
1	1	0	1	1	No	
0	1	1	0	0	Yes	17
1	1	1	0	0	Yes	19
0	1	1	1	0	Yes	21
1	1	1	1	0	No	

one. Furthermore, at any node, a specification of the values of all the free decision variables is called a *completion* of the node.

The technique then determines whether the node can be fathomed by applying the following three principles:

1. If completing a node by setting each free decision variable equal to one, which makes the objective function largest, results in a feasible solution; further branching on the node is unnecessary since the solution is certainly the best feasible completion of the node.
2. If the best completion of the node is unfeasible, it still provides an upper bound on the best objective function values that can be obtained via a feasible solution of the node. This bound eliminates the node from consideration if a previously found candidate solution has a higher objective function value.
3. If assigning the value zero to the free variables does not satisfy a constraint, then no completion of the node can satisfy the constraint and further branching on the node is unnecessary since all completions of the node are unfeasible.

If the technique fails to obtain any information about a node, it branches on a free decision variable and adds two new nodes. These new nodes are then analyzed as explained above. A numerical illustration based on the problem

$$\begin{array}{ll} \text{Max} & Z = 2x_1 + 12x_2 + 5x_3 + 4x_4 + 8x_5 \\ \text{Subject to} & 2x_1 + 4x_2 + 2x_3 + 3x_4 + 4x_5 \leq 9 \\ & x_1 + x_2 + x_3 + x_4 + x_5 \leq 3 \\ & x_1, x_2, x_3, x_4, x_5 = 0,1 \end{array}$$

is presented in figure 13.

The technique first branches on free variable x_1 and fails to eliminate node 2. It then branches on free variable x_2, but again fails to eliminate node 4, so it branches one more time on free variable x_3. Since the three fixed variables x_1, x_2 and x_3 already make the second constraint binding, node 6 is fathomed and the technique continues examining node 7. It should be noted that node 5 is

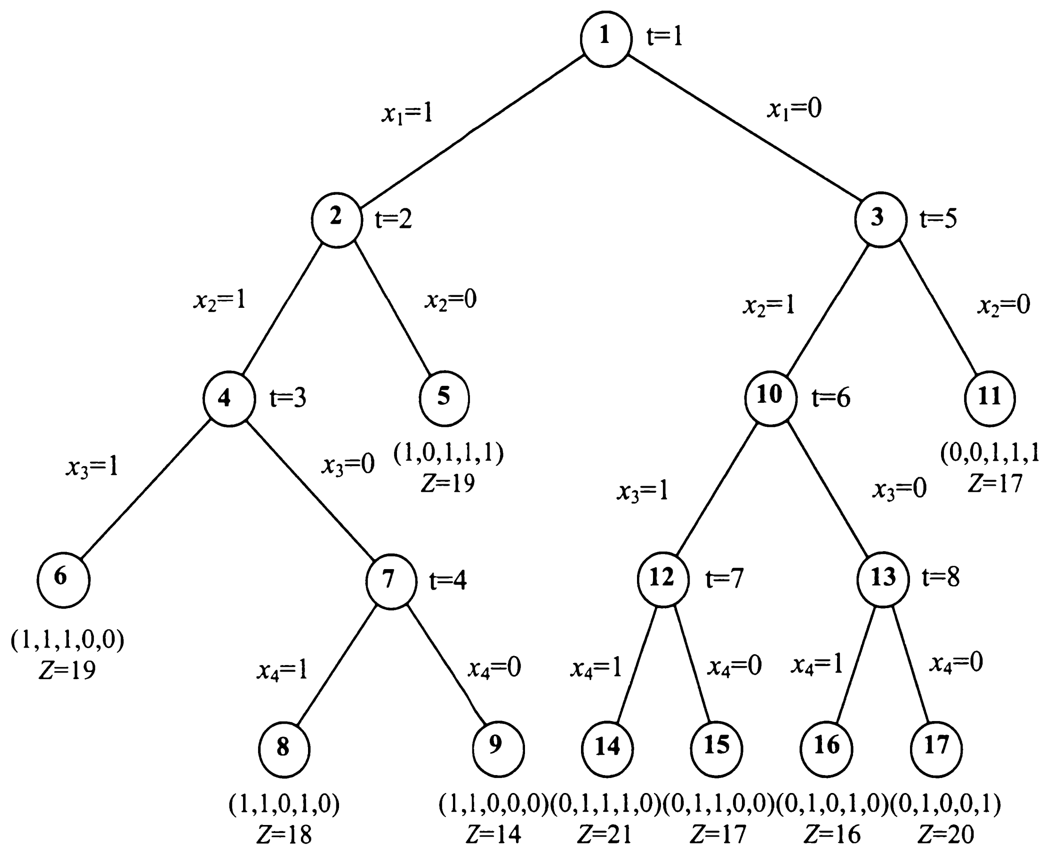


Figure 13. Search Enumeration Technique's Numerical Example.

fathomed since its best completion is unfeasible with an objective function value equal to that of feasible solution (1,1,1,0,0) under node 6. Furthermore, node 11 is fathomed since its best completion is feasible with a lower-than-already found objective function value. Finally, only 14 out of the 32 (2^5) possible solutions were evaluated.

It must be emphasized that, unlike the previous numerical example where all variables were free at the beginning, only non-basic decision variables in the original unfeasible solution become free variables when the search enumeration technique is applied to the EMP models since the original basic decision variables' values are fixed during the forward step of the local search method. Moreover, if the decision variables are sorted in descending order according to their objective function coefficient, the search enumeration technique is likely to evaluate fewer possible solutions. In fact, if free variables x_2, x_5, x_3, x_4 and x_1 in the numerical example had been made fixed variables in this order, the search enumeration technique would have evaluated only 10 possible solutions. This is due to the fact that, by fixing variables with larger objective function coefficients first, there is a better chance of establishing higher upper bounds earlier.

4.4 Numerical Methods to Minimize Marginal Energy Consumption

4.4.1 Controlled Marginal Energy Consumption Method

The algorithm for this method, which allows throughput to increase as much as possible while limiting additional energy consumption and is presented in figure 14, first separates the operational strategies into final-process and intermediate-process due to their differing energy impacts. The algorithm then ranks intermediate-process strategies according to the decreasing PBC values of the processes to which they apply. Finally, the algorithm progressively selects strategies from the following groups using the greedy concept, which chooses at each step the strategy bringing the "best" immediate reward (i.e., the greatest increase in total throughput per dollar invested):

1. Final-process strategies: the best immediate reward is measured by the ratio ${\Delta P_{j(a)}}/{I_a}$.

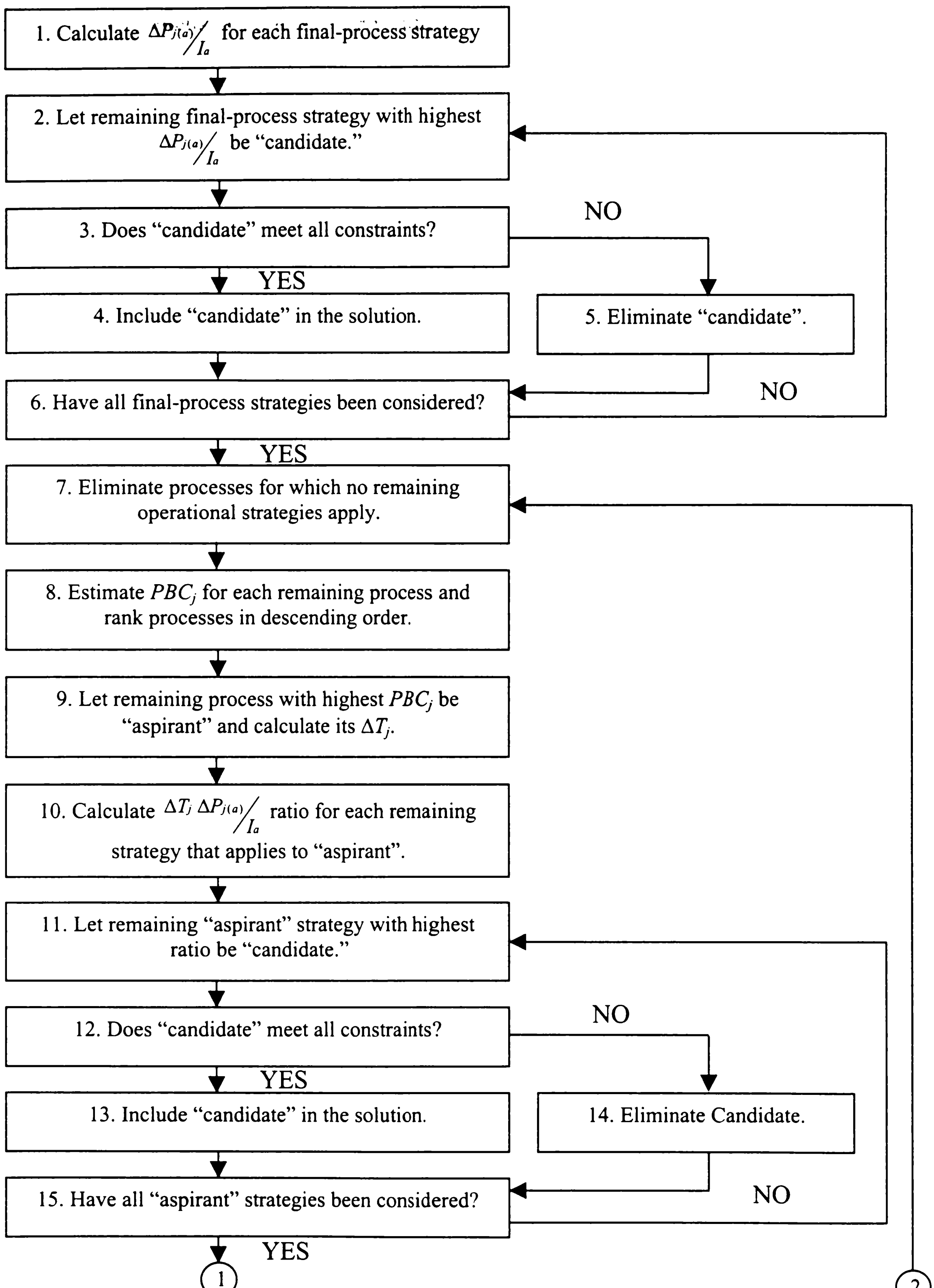


Figure 14. Algorithm for Controlled Marginal Energy Consumption Method.

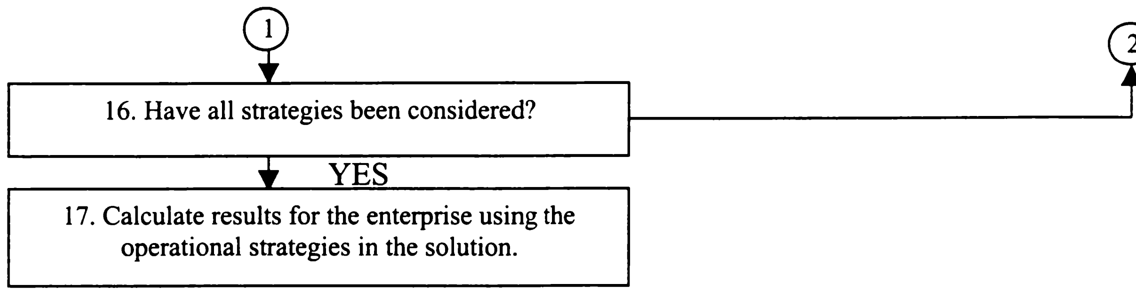


Figure 14. Continued.

2. Intermediate-process strategies with the same PBC value and starting with the highest: the best immediate reward is measured by the ratio $\Delta T_j \, \Delta P_{j(a)} / I_a$.

The algorithm guarantees feasibility by rejecting any strategy whose selection will cause a violation of the upper-bound, capacity or budget constraints. Moreover, the algorithm recalculates upstream PBC values after each strategy is selected.

4.4.2 Pareto Optimal Solution Method

The Pareto optimal solution method is a solution strategy applicable to multi-objective decision making. Therefore, a solution *A* is said to be Pareto optimal if no other feasible solution is at least as good as *A* with respect to every objective and strictly better than *A* with respect to at least one objective. A Pareto optimal solution can also be defined in terms of domination: a feasible solution *A* dominates a feasible solution *B* if *A* is at least as good as *B* with respect to every objective and strictly better than *B* with respect to at least one objective. In other words, a Pareto optimal solution is a feasible solution that is not dominated. Moreover, all the Pareto optimal solutions for a given multiple-objective problem form a trade-off curve or efficient frontier. Although a trade-off curve does not specify the best solution, it is useful because it gives the ultimate decision maker many solutions to choose from, none of which is dominated by any others (Winston and Albright, 2001).

In light of the previous definitions, the solution found through the controlled marginal energy consumption method described before is actually a Pareto optimal solution: there is no other feasible solution to the problem that will generate as much additional throughput while requiring less additional energy. Accordingly, this solution is somewhere on the trade-off curve in figure 15, where V_{st} represents the t^{th} value for the s^{th} objective function. The construction of the trade-off curve is accomplished by parametrically varying specified levels of all but one objective while repeatedly optimizing the other (Rardin, 1998). This procedure will be explained next for the four points in the trade-off curve of figure 15. It is important to note that points (V_{11}, V_{21}) and (V_{14}, V_{24}) are the extreme points of the curve since a feasible solution will neither consume less additional energy than zero nor more energy than *Em*.

To calculate the point (V_{11}, V_{21}), we will first set V_{11} equal to zero – the best value for objective 1. Then, V_{21} becomes the maximum ΔT for given a B, which is calculated using steps 1 through 6 in figure 14's algorithm since only final-process operational strategies guarantee no additional energy consumption. Obviously, if the budget is exhausted before the algorithm gets to step 7, the

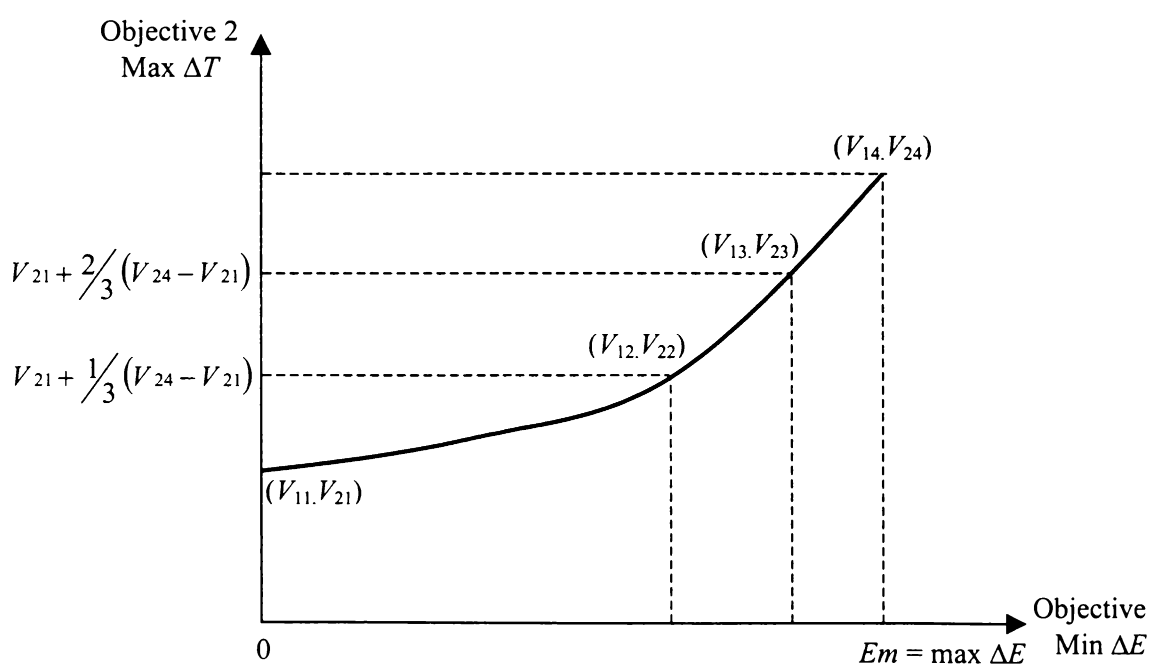


Figure 15. Trade-off Curve for the Increase Throughput and Minimize the Marginal Energy Consumption Increase Problem.

solution for the controlled marginal energy consumption method is exactly point (V_{11},V_{21}) and the trade-off curve only contains that point.

To calculate the point (V_{14},V_{24}), we will ignore objective 1 to obtain the best value for objective 2. In other words, V_{24} becomes the maximum ΔT for given a B but without the energy restriction, which can be worked out by using the algorithm in figure 16, where the strategies are not categorized to be selected based on same-energy-impact groups as is the case in figure 14. Once the value for V_{24} is determined, the EM model is updated to obtain V_{14}.

To calculate points (V_{12},V_{22}) and (V_{13},V_{23}), we first compute

$$V_{22} = V_{21} + \frac{1}{3}\left(V_{24} - V_{21}\right)$$

$$V_{23} = V_{21} + \frac{2}{3}\left(V_{24} - V_{21}\right)$$

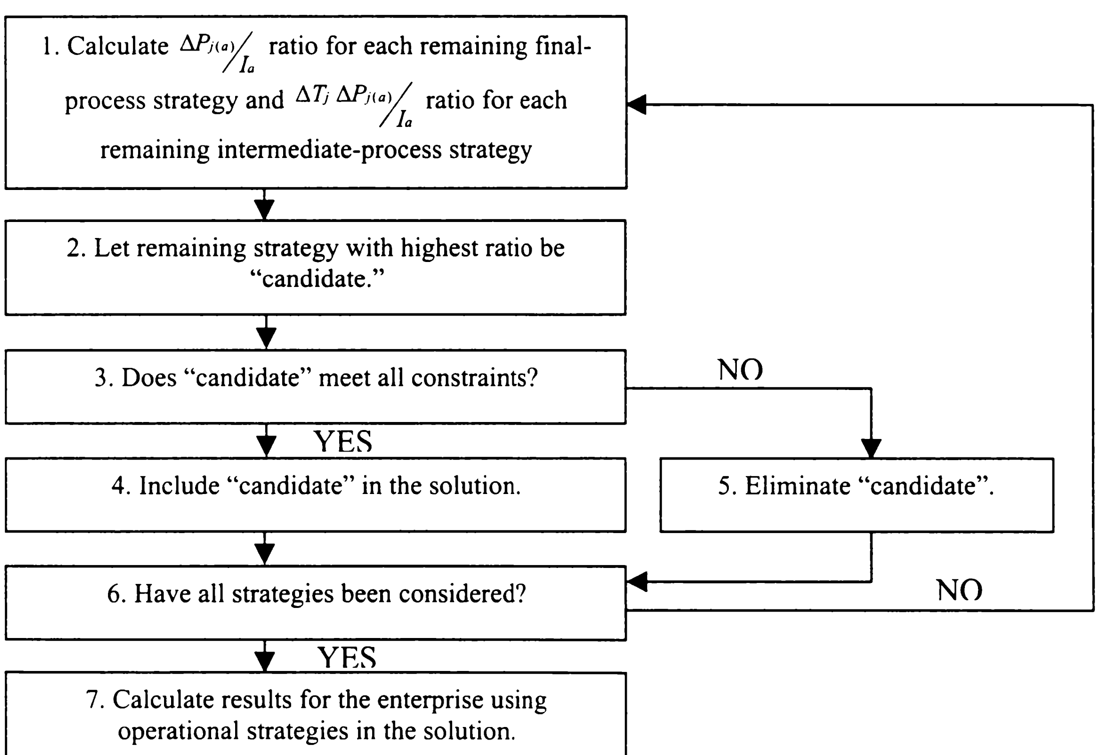


Figure 16. Algorithm for Maximizing Total Throughput.

Next, the algorithm in figure 14 is used with the following condition added to steps 4 and 13: if, after the inclusion of the "candidate," the solution forces ΔT above either V_{22} or V_{23}; the algorithm stops. Then, fractional benefits are calculated for the last operational strategy added to the solution so that exact V_{22} or V_{23} values are obtained, and the EM model is used to compute V_{12} and V_{13} values.

4.5 Numerical Methods to Minimize Energy Consumption

Since the EMP model to maintain throughput and minimize total energy consumption does not exhibit nonlinear capacity constraints, a simplified version of the numerical method to maximize profit will be used as presented in figure 17. Likewise, the greedy algorithm for this method, which is based on the ratio ${}^{PB_j}\!/_{I_a}$ is shown in figure 18.

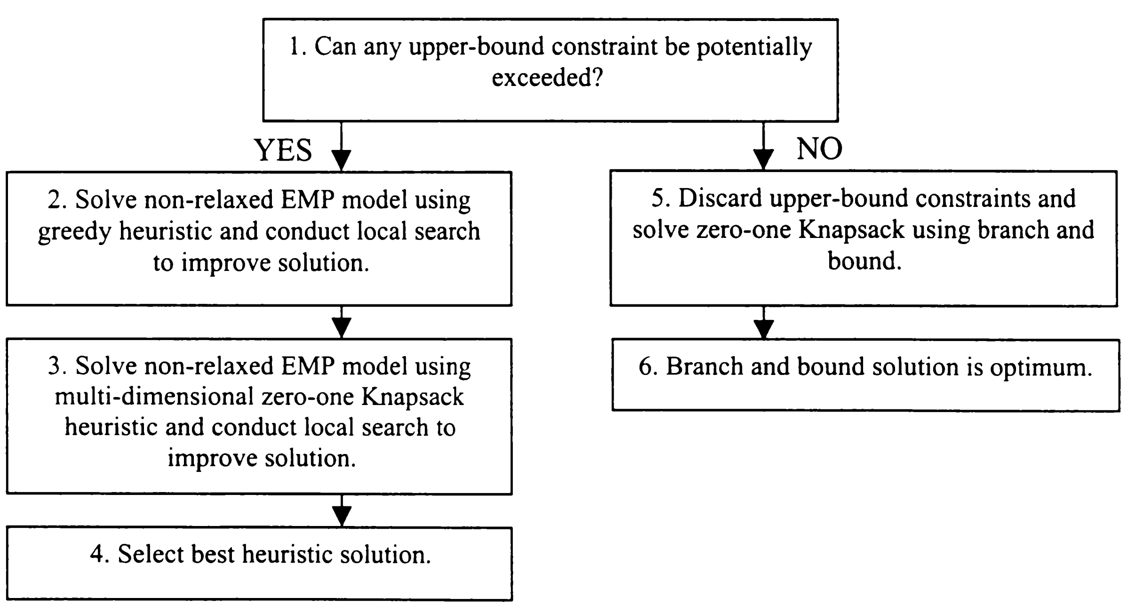


Figure 17. Numerical Method for Energy Consumption Minimization.

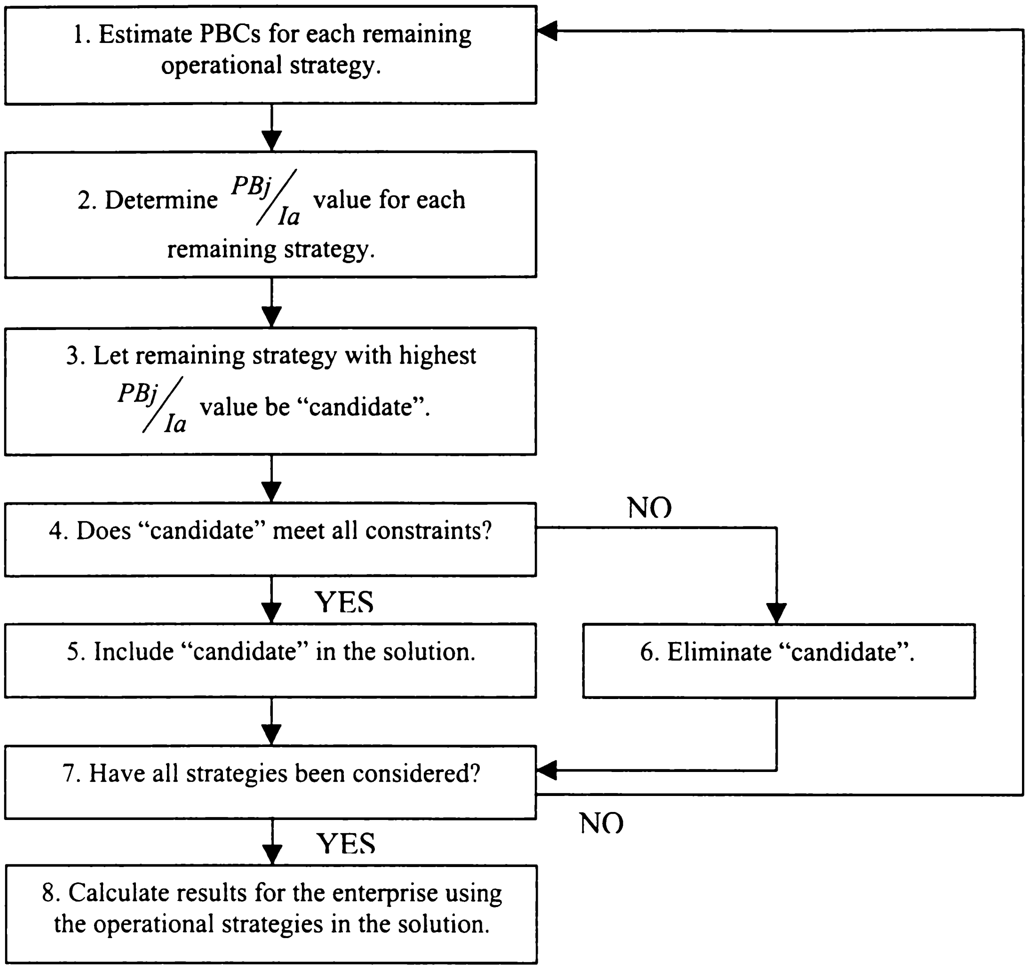


Figure 18. Greedy Algorithm for Energy Consumption Minimization.

4.6 Comprehensive Numerical Examples

Two comprehensive numerical examples are presented next to illustrate the solution methodologies for both maximize-profit and minimize-marginal-energy-consumption philosophies. No numerical example for the minimize-energy-consumption philosophy is included since its solution methodology is a simplified version of the proposed numerical method to maximize profit as explained in section 4.5.

4.6.1. Numerical Example for Maximize-Profit Philosophy

The following numerical examples are based on the hypothetical production process in figure 19. Its Type 2 EM model is presented in table 14, where input variables are shown in bold font style – the cost per ton of raw material, C_0, is assumed to be equal to \$25. Moreover, the hypothetical operational strategies indicators are displayed in table 15. Finally, to increase the understanding of the results obtained by the solution methodologies, the optimal solution, along with the next seven top solutions, for the increase throughput and maximize profit philosophy is presented in table 16.

If the maximum processing capacity for each process is

$$K_1 = 11{,}000$$
$$K_2 = 10{,}750$$
$$K_3 = 10{,}500$$
$$K_4 = 10{,}000$$
$$K_5 = 5{,}000$$
$$K_6 = 5{,}000$$
$$K_7 = 5{,}000$$
$$K_8 = 3{,}300$$
$$K_9 = 3{,}300$$

The capacity constraints for this example are

$$\widetilde{P}_1 \leq 299$$
$$\widetilde{P}_2 \leq 370$$
$$\widetilde{P}_3 \leq 327$$
$$\widetilde{P}_4 \leq 336$$

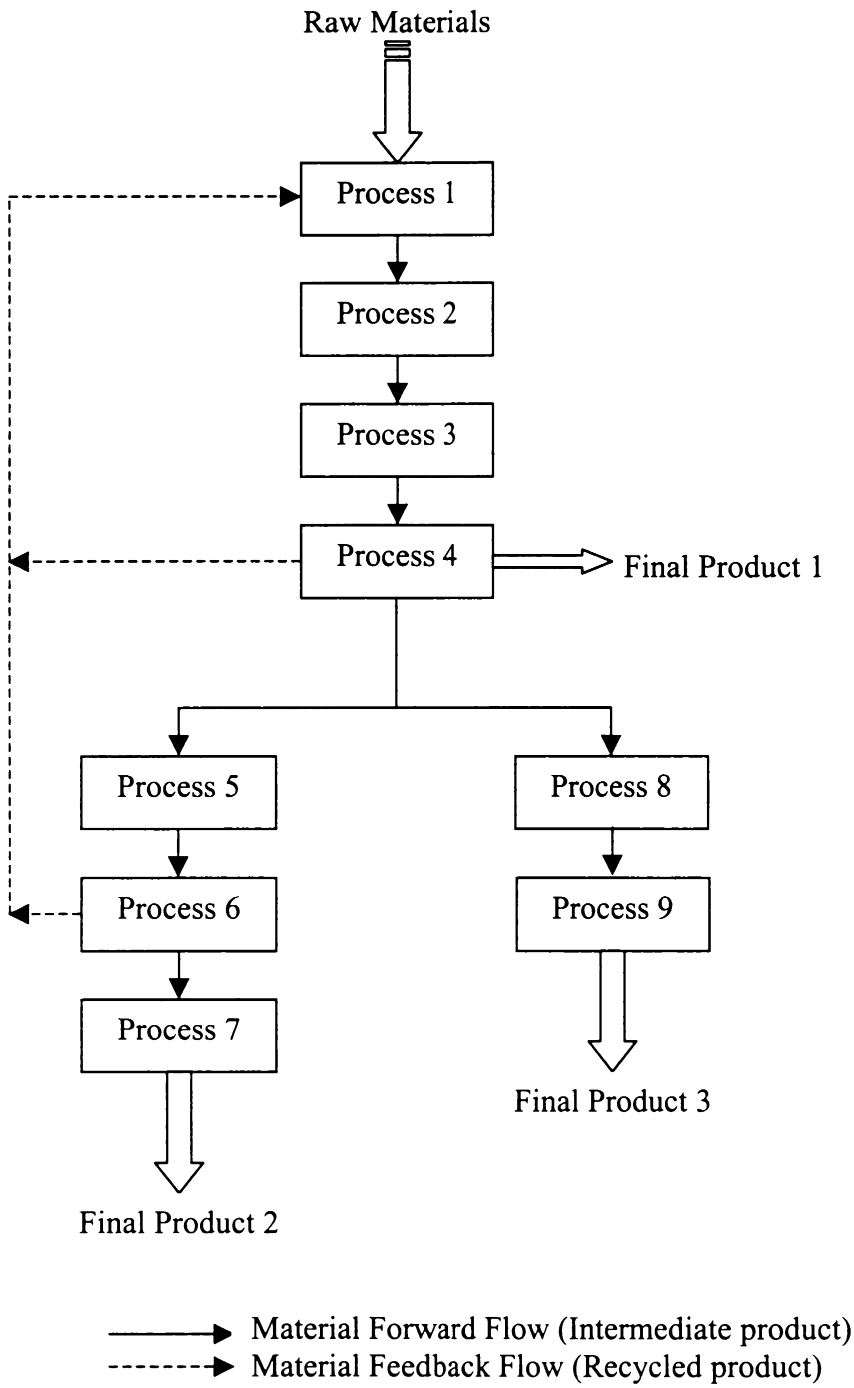


Figure 19. Hypothetical Production Process for the Comprehensive Examples

Table 14. Type 2 EM Model's Comprehensive Numerical Example

			TO PROCESS									*P* Totals		TO FINAL PRODUCT			Totals	Material loss %	Material input
			1	2	3	4	5	6	7	8	9			1	2	3			
FROM PROCESS	0	*P*	**10,000**									10,000	*T*						
		r	**1.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**		*r*	**0.000**	**0.000**	**0.000**	1.000	0.000	
	1	*P*		10,380								10,380	*T*				10,380		
		r	**0.000**	**0.970**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**		*r*	**0.000**	**0.000**	**0.000**	0.970	0.030	10,701
	2	*P*			10,173							10,173	*T*				10,173		
		r	**0.000**	**0.000**	**0.980**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**		*r*	**0.000**	**0.000**	**0.000**	0.980	0.020	
	3	*P*				9,664						9,664	*T*				9,664		
		r	**0.000**	**0.000**	**0.000**	**0.950**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**		*r*	**0.000**	**0.000**	**0.000**	0.950	0.050	
	4	*P*	483				4,542			3,189		8,214	*T*	966			9,181		
		r	**0.050**	**0.000**	**0.000**	**0.000**	**0.470**	**0.000**	**0.000**	**0.330**	**0.000**		*r*	**0.100**	**0.000**	**0.000**	0.950	0.050	
	5	*P*						4,360				4,360	*T*				4,360		
		r	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.960**	**0.000**	**0.000**	**0.000**		*r*	**0.000**	**0.000**	**0.000**	0.960	0.040	
	6	*P*	218						4,055			4,273	*T*				4,273		
		r	**0.050**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.930**	**0.000**	**0.000**		*r*	**0.000**	**0.000**	**0.000**	0.980	0.020	
	7	*P*										0	*T*		3,933		3,933		
		r	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**		*r*	**0.000**	**0.970**	**0.000**	0.970	0.030	
	8	*P*									2,998	2,998	*T*				2,998		
		r	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.940**		*r*	**0.000**	**0.000**	**0.000**	0.940	0.060	
	9	*P*										0	*T*			2,758	2,758		
		r	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**		*r*	**0.000**	**0.000**	**0.920**	0.920	0.080	
P Totals			10,701	10,380	10,173	9,664	4,542	4,360	4,055	3,189	2,998		*T*	966	3,933	2,758	7,658		
Yield			0.970	0.980	0.950	0.900	0.960	0.930	0.970	0.940	0.920								
Energy/ton			**20**	**16**	**22**	**18**	**22**	**15**	**21**	**19**	**21**								
Energy			214,024	166,083	223,797	173,951	99,925	65,406	85,158	60,593	62,953	1,151,890							
Fixed cost/ton			**7.0**	**10.0**	**10.0**	**4.0**	**8.0**	**4.0**	**5.0**	**5.0**	**3.0**								
Var. cost/ton			**5.0**	**8.0**	**4.0**	**8.0**	**6.0**	**7.0**	**3.0**	**2.0**	**5.0**								
Acc. cost/ton			43.8	63.0	81.1	98.0	116.6	130.2	142.5	111.7	130.1								
Revenue/ton														**108.0**	**155.0**	**140.0**			
Profit														9,700	49,139	27,393	86,232		

Table 15. Hypothetical Operational Strategies' Indicators

OPERATIONAL STRATEGY	$\Delta P^{A}_{j(a)}$	$\Delta P^{B}_{j(a)}$	I_a	AE_a
OS(1,1)	130		2,075	600
OS(2,2)	100		2,300	500
OS(3,3)	150		5,150	1,535
OS(4,4)	125		4,800	1,110
OS(5,5)	120		4,950	1,065
OS(5,6)	90		3,550	735
OS(6,7)		45	600	175
OS(7,8)	50		2,475	675
OS(8,9)	65		3,075	855
OS(9,10)	75		3,625	955

Table 16. Top Eight Solutions for the Increase Throughput, Maximize Profit Philosophy

	OPERATIONAL STRATEGIES [1]										
	1	2	3	4	5	6	7	8	9	10	AW
1	0	0	0	0	1	1	0	1	0	1	38,708
2	0	1	0	0	1	1	0	0	0	1	37,811
3	0	1	0	1	1	0	0	1	0	0	37,588
4	1	0	0	0	1	1	0	0	0	1	37,331
5	1	1	0	0	0	1	0	1	0	1	37,323
6	0	0	0	1	0	1	0	1	0	1	37,298
7	1	0	0	1	1	0	0	1	0	0	37,109
8	0	1	0	0	1	1	1	0	1	0	36,856

[1] "1" indicates that an operational strategy is included in the solution

$$\widetilde{P}_5 \leq 450$$
$$\widetilde{P}_6 \leq 640$$
$$\widetilde{P}_7 \leq 945$$
$$\widetilde{P}_8 \leq 111$$
$$\widetilde{P}_9 \leq 302$$

and the budget constraint takes the form

$$\sum_{a=1}^{10}\left[I_a \, OS(j,a)\right] \leq 14{,}600$$

4.6.1.1 Numerical Example for Non-Relaxed EMP Model Heuristic

The first step in the numerical method for profit maximization –see figure 8- is to solve the non-relaxed EMP model using the heuristic explained in section 4.3.1.

The results of the first iteration, which are shown in figure 20 and are similar in format to those in section 4.3.1.1's example, indicate that *OS*(5,6) should be selected. The second iteration selects *OS*(7,8). Then, part A of the third iteration attempts to choose *OS*(5,5), but its inclusion in the solution violates the upper-bound constraint for the 5th process and it's discarded. Consequently, the heuristic picks *OS*(1,1) during the part B of the third iteration. In the fourth and fifth iteration, *OS*(2,2) and *OS*(9,10) are selected, respectively. At this point, the algorithm stops since the budget constraint will not allow any additional remaining strategies from being included.

The heuristic's solution is then {*OS*(1,1), *OS*(2,2), *OS*(5,6), *OS*(7,8), *OS*(9,10)}, which according to table 16 is suboptimal with an objective value of $37,323.

4.6.1.2 Numerical Example for Local Search Method to Improve Solution

A local search is part of the first step in figure 8 in an attempt to improve on a possibly suboptimal heuristic solution.

The current solution x^{now} is represented by the 10-element vector (1,1,0,0,0,1,0,1,0,1), which means that *OS*(1,1) = 1, *OS*(2,2) = 1, *OS*(3,3) = 0, *OS*(4,4) = 0 and so on.

FIRST ITERATION

OPERATIONAL STRATEGIES		
a	AW_a	Index
1	5,182	2.497
2	5,766	2.507
3	10,329	2.006
4	11,295	2.353
5	12,906	2.607
6	**9,768**	**2.752**
7	1,247	2.078
8	6,672	2.696
9	6,497	2.113
10	8,955	2.470

CONSTRAINTS			
j	$\hat{Y}_j$	$\widetilde{P}_j$	$\sum I_a$
1	0.970	4.8	
2	0.980	4.7	
3	0.950	4.6	
4	0.900	4.4	
5	0.980	2.0	
6	0.930	92.0	
7	0.970	85.6	
8	0.940	1.4	
9	0.920	1.3	
		Total	3,550

SECOND ITERATION

OPERATIONAL STRATEGIES		
a	AW_a	Index
1	5,334	2.571
2	5,886	2.559
3	10,519	2.043
4	11,464	2.388
5	12,919	2.610
6		
7	1,331	2.218
8	**6,836**	**2.762**
9	6,500	2.114
10	8,960	2.472

CONSTRAINTS			
j	$\hat{Y}_j$	$\widetilde{P}_j$	$\sum I_a$
1	0.970	4.8	
2	0.980	4.7	
3	0.950	4.6	
4	0.900	4.4	
5	0.980	2.0	
6	0.930	92.0	
7	0.982	85.6	
8	0.940	1.4	
9	0.920	1.3	
		Total	6,025

THIRD ITERATION (Part A)

OPERATIONAL STRATEGIES		
a	AW_a	Index
1	5,440	2.622
2	5,970	2.596
3	10,652	2.068
4	11,580	2.413
5	**13,138**	**2.654**
6		
7	1,367	2.278
8		
9	6,501	2.114
10	8,960	2.472

CONSTRAINTS			
j	$\hat{Y}_j$	$\widetilde{P}_j$	$\sum I_a$
1	0.970	11.2	
2	0.980	10.9	
3	0.950	10.7	
4	0.900	10.2	
5	**1.006**	4.8	
6	0.930	214.8	
7	0.982	199.8	
8	0.940	3.4	
9	0.920	3.2	
		Total	10,975

Figure 20. Non-Realxed Heuristic's Comprehensive Numerical Example Worksheets.

THIRD ITERATION (Part B)

OPERATIONAL STRATEGIES		
a	AW_a	Index
1	**5,440**	**2.622**
2	5,970	2.596
3	10,652	2.068
4	11,580	2.413
5		
6		
7	1,367	2.278
8		
9	6,501	2.114
10	8,960	2.472

CONSTRAINTS			
j	$\hat{Y}_j$	$\widetilde{P}_j$	$\sum I_a$
1	0.982	14.3	
2	0.980	144.0	
3	0.950	141.2	
4	0.900	134.1	
5	0.980	63.0	
6	0.930	151.8	
7	0.982	141.1	
8	0.940	44.3	
9	0.920	41.6	
		Total	8,100

FOURTH ITERATION

OPERATIONAL STRATEGIES		
a	AW_a	Index
1		
2	**6,066**	**2.637**
3	10,833	2.103
4	11,763	2.451
5		
6		
7	1,419	2.365
8		
9	6,605	2.148
10	9,101	2.511

CONSTRAINTS			
j	$\hat{Y}_j$	$\widetilde{P}_j$	$\sum I_a$
1	0.982	21.8	
2	0.990	151.4	
3	0.950	249.9	
4	0.900	237.4	
5	0.980	111.6	
6	0.930	199.3	
7	0.982	185.4	
8	0.940	78.3	
9	0.920	73.6	
		Total	10,400

FIFTH ITERATION

OPERATIONAL STRATEGIES		
a	AW_a	Index
1		
2		
3	10,980	2.132
4	11,909	2.481
5		
6		
7	1,469	2.448
8		
9	6,690	2.176
10	**9,213**	**2.542**

CONSTRAINTS			
j	$\hat{Y}_j$	$\widetilde{P}_j$	$\sum I_a$
1	0.982	21.8	
2	0.990	151.4	
3	0.950	249.9	
4	0.900	237.4	
5	0.980	111.6	
6	0.930	199.3	
7	0.982	185.4	
8	0.940	78.3	
9	0.945	73.6	
		Total	14,025

Figure 20. Continued.

Iteration 1, step 1

Set $x^{best} = x^{now}$; $f^{best} = 37{,}323$; and $H = \{\ \}$.

Iteration 1, step 2

Build neighborhood $N(H, x^{now})$ by switching, one by one, each basic decision variable in the solution from one to zero; and by rebuilding all possible feasible solutions through the addition of non-basic decision variables as presented in the worksheet below for the five current basic decision variables: *OS*(1,1), *OS*(2,2), *OS*(5,6), *OS*(7,8) and *OS*(9,10).

		OPERATIONAL STRATEGIES										AW_a
		1	2	3	4	5	6	7	8	9	10	
x^{now}	I	1	1	0	0	0	1	0	1	0	1	37,323
OS(1,1)=0	II	0	1	0	0	0	1	0	1	0	1	31,644
	III	0	1	0	0	0	1	1	1	0	1	33,110
OS(2,2)=0	IV	1	0	0	0	0	1	0	1	0	1	31,114
	V	1	0	0	0	0	1	1	1	0	1	32,613
OS(5,6)=0	VI	1	1	0	0	0	0	0	1	0	1	27,109
	VII	1	1	0	0	0	0	1	1	1	1	35,496
OS(7,8)=0	VIII	1	1	0	0	0	1	0	0	0	1	30,296
	IX	1	1	0	0	0	1	1	0	0	1	31,778
OS(9,10)=0	X	1	1	0	0	0	1	0	1	0	0	28,110
	XI	1	1	0	0	0	1	1	1	1	0	36,307

Row I in the worksheet above represents x^{now}. In turn, rows II, IV, VI, VIII and X show the resulting solution when one of the basic variables is made equal to zero as indicated. Then, rows III, V, VII, IX and XI display the best feasible completion after fixing the values of the original basic variables. For instance, in row III, *OS*(5,5) must be equal to zero to prevent an upper-bound constraint violation since *OS*(5,6) is already equal to one. Moreover, *OS*(3,3), *OS*(4,4) and *OS*(8,9) cannot be included because any would cause a budget violation. As a result, making *OS*(6,7) equal to one is the best feasible completion.

Since there is no $x^{next} \in N(H,x^{now})$ such that $f(x^{next}) - f(x^{now}) > 0$, the search is diversified by choosing a $x^{next} \in N(H,x^{now})$ such that $[f(x^{now}) - f(x^{next})]$ is smallest. In other words, x^{next} becomes (1,1,0,0,0,1,1,1,1,0) since it exhibits the highest AW of all possible moves.

Iteration 1, step 3

Set $x^{now} = x^{next}$ and

$$H = \begin{Bmatrix} OS(6,7) \text{ cannot change from 1 to 0 until iteration 8} \\ OS(8,9) \text{ cannot change from 1 to 0 until iteration 8} \\ OS(9,10) \text{ cannot change from 0 to 1 until iteration 8} \end{Bmatrix}$$

if variable *tabutenure* is equal to 7.

Iteration 2, step 2

The worksheet below presents the creation of the new neighborhood $N(H, x^{now})$. As before, rows II, III, IV and V show the resulting solution when one of the basic variables is made equal to zero as indicated. However, no non-basic variables can be added to those solutions to create feasible solutions, which results in an empty $N(H, x^{now})$. The local search method stops at this point and fails to improve the heuristic solution.

		OPERATIONAL STRATEGIES										AW_a
		1	2	3	4	5	6	7	8	9	10	
x^{now}	I	1	1	0	0	0	1	1	1	1	0	36,307
$OS(1,1)=0$	II	0	1	0	0	0	1	1	1	1	0	30,610
$OS(2,2)=0$	III	1	0	0	0	0	1	1	1	1	0	30,106
$OS(5,6)=0$	IV	1	1	0	0	0	0	1	1	1	0	26,004
$OS(7,8)=0$	X	1	1	0	0	0	1	1	0	1	0	29,242

Obviously, if the solutions in the worksheet above are allowed to be part of $N(H, x^{now})$, which means that is not required to add non-basic variables to build the neighborhood, and since there is no $x^{next} \in N(H,x^{now})$ such that $f(x^{next}) - f(x^{now}) > 0$, the search would be diversified again by making x^{next} equal to (0,1,0,0,0,1,1,1,1,0) since it would exhibit the highest AW of all possible moves.

4.6.1.3 Numerical Example for Multi-Dimensional Zero-One Knapsack Heuristic

Given the fact that the upper-bound constraint for the fifth process in figure 17 can be exceeded through the selection of both $OS(5,5)$ and $OS(5,6)$, the next step in figure 8's solution method is to relax the capacity constraints and solve the resulting multi-dimensional zero-one Knapsack problem as follows.

Iteration 1, step 1

Set

$J = \{OS(1,1), OS(2,2), OS(3,3), OS(4,4), OS(5,5), OS(5,6), OS(6,7), OS(7,8), OS(8,9), OS(9,10)\}$

$T = \{OS(1,1), OS(2,2), OS(3,3), OS(4,4), OS(5,5), OS(5,6), OS(6,7), OS(7,8), OS(8,9), OS(9,10)\}$

$S = \{\ \}$

$R = \{\ \}$

$N = (1;1;1;1;1;1;1;1;1)$

$Z = 0$

Set $d_{j,a}$ values as presented in matrix below; where j is row and a is column, and D_1 is column 1, D_2 is column 2, and so on,

$$d_{j,a} = \begin{bmatrix} 0.405 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.482 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.295 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.129 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.660 & 0.495 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.411 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.340 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.313 \\ 0.142 & 0.158 & 0.353 & 0.329 & 0.339 & 0.243 & 0.041 & 0.170 & 0.211 & 0.248 \end{bmatrix}$$

Iteration 1, step 2

Set $U = \{OS(1,1), OS(2,2), OS(3,3), OS(4,4), OS(5,5), OS(5,6), OS(6,7), OS(7,8), OS(8,9), OS(9,10)\}$

Iteration 1, step 3

Calculate gradients g_a given that $R = \{\ \}$ and $m = 10$ as presented in the worksheet below,

a	1	2	3	4	5	6	7	8	9	10
AW_a	5,182	5,766	10,329	11,295	12,906	9,768	1,247	6,672	6,497	8,955
g_a	29,954	28,524	50,433	77,967	40,831	41,825	**95,955**	36,344	37,333	50,476

Iteration 1, step 4

Set S = {OS(6,7)}

$T = \{OS(1,1), OS(2,2), OS(3,3), OS(4,4), OS(5,5), OS(5,6), OS(7,8), OS(8,9), OS(9,10)\}$

$R = \{D_7\}$

Z = 1,247

Iteration 2, step 2

Set U = $\{OS(1,1), OS(2,2), OS(3,3), OS(4,4), OS(5,5), OS(5,6), OS(7,8), OS(8,9), OS(9,10)\}$

Iteration 2, step 3

Calculate gradients g_a given that $R = \{D_7\}$ and $m = 10$ as presented in the worksheet below,

a	1	2	3	**4**	5	6	7	8	9	10
AW_a	5,230	5,813	10,413	11,378	13,018	9,851		6,707	6,532	9,002
g_a	29,146	26,682	32,081	**40,891**	22,897	23,612		32,883	29,996	37,156

Iteration 2, step 4

Set S = {OS(6,7), OS(4,4)}

$T = \{OS(1,1), OS(2,2), OS(3,3), OS(5,5), OS(5,6), OS(7,8), OS(8,9), OS(9,10)\}$

$R = \{D_4, D_7\}$

Z = 12,625

Iteration 3, step 2

Set U = $\{OS(1,1), OS(2,2), OS(3,3), OS(5,5), OS(5,6), OS(7,8), OS(8,9), OS(9,10)\}$

Iteration 3, step 3

Calculate gradients g_a given that $R = \{D_4, D_7\}$ and $m = 10$ as presented in the worksheet below,

a	1	2	3	4	5	6	7	8	9	**10**
AW_a	5,412	5,956	10,640		13,247	10,024		6,822	6,648	9,158
g_a	30,061	26,992	34,053		22,614	23,277		33,517	31,180	**38,921**

Iteration 3, step 4

Set $S = \{OS(6,7), OS(4,4), OS(9,10)\}$,

$T = \{OS(1,1), OS(2,2), OS(3,3), OS(5,5), OS(5,6), OS(7,8), OS(8,9)\}$

$R = \{D_4, D_7, D_{10}\}$

$Z = 21,783$

Iteration 4, step 2

Set U = $\{OS(1,1), OS(2,2), OS(3,3), OS(5,5), OS(5,6), OS(7,8), OS(8,9)\}$

Iteration 4, step 3

Calculate gradients g_a given that $R = \{ D_4, D_7, D_{10}\}$ and $m = 10$ as presented in the worksheet below,

a	1	2	**3**	4	5	6	7	8	9	10
AW_a	5,556	6,069	10,819		13,255	10,030		6,822	6,880	
g_a	30,647	27,081	**35,765**		21,945	22,553		33,449	32,783	

Iteration 4, step 4

Set $S = \{OS(6,7), OS(4,4), OS(9,10), OS(3,3)\}$,

$T = \{OS(1,1), OS(2,2), OS(5,5), OS(5,6), OS(7,8), OS(8,9)\}$

$R = \{D_3, D_4, D_7, D_{10}\}$

$Z = 32,602$

Iteration 5, step 2

Since no strategy in T will fit within the available remaining budget, the algorithm sets U = { } and stops.

It is important to note that the resulting heuristic solution, whose objective function value is \$32,602, is not feasible: it violates the capacity constraint of the 8th process.

4.6.1.4 Numerical Example for Local Search Method to Find Feasible Solution

A local search is part of the seventh step in figure 8 in an attempt to move from an infeasible solution, such as the one found in the previous section, to a feasible one.

The current infeasible solution is represented by (0,0,1,1,0,0,1,0,0,1); and the local search looks for feasible solutions around it by making basic decision variables –in this case *OS*(3,3), *OS*(4,4), *OS*(6,7) and *OS*(9,10)- equal to zero one at a time, and then adding non-basic variables based on a search enumeration technique as explained next. The worksheet below, whose format is identical to the ones in section 4.6.1.2, illustrates the two-step procedure.

		OPERATIONAL STRATEGIES										AW_a
		1	2	3	4	5	6	7	8	9	10	
Current	I	0	0	1	1	0	0	1	0	0	1	32,602
OS(3,3)=0	II	0	0	0	1	0	0	1	0	0	1	21,783
	III	1	0	0	1	0	0	1	1	0	1	34,268
	IV	1	0	0	1	0	0	1	0	1	1	34,331
	V	0	1	0	1	0	0	1	1	0	1	34,758
	VI	0	1	0	1	0	0	1	0	1	1	34,820
	VII	0	0	0	1	1	0	1	0	0	1	35,039
	VIII	0	0	0	1	0	1	1	0	0	1	31,813
	IX	0	0	0	1	0	0	1	1	1	1	35,486
OS(4,4)=0	X	0	0	1	0	0	0	1	0	0	1	20,839
	XI	1	0	1	0	0	0	1	0	0	1	Infeasible
	XII	0	1	1	0	0	0	1	1	0	1	33,830
	XIII	0	0	1	0	1	0	1	0	0	1	34,123
	XIV	0	0	1	0	0	1	1	0	0	1	30,890
	XV	0	0	1	0	0	0	1	1	0	1	27,676
	XVI	0	0	1	0	0	0	1	0	1	1	27,735
OS(6,7)=0	XVII	0	0	1	1	0	0	0	0	0	1	31,134
OS(9,10)=0	XVIII	0	0	1	1	0	0	1	0	0	0	Infeasible

Row I in the worksheet above represents the current infeasible solution. In turn, rows II, X, XVII and XVIII show the resulting solution during the back step when one of the basic variables is made equal to zero as indicated. The other rows display the best feasible solution using the search tree technique and after fixing the values of the original basic variables.

For example, in row III, variables $OS(3,3)$, $OS(4,4)$, $OS(6,7)$ and $OS(9,10)$ are fixed; and the method branches on free variable $OS(1,1)$. When $OS(1,1)$ is first fixed with a value of one, $OS(2,2)$, $OS(5,5)$ and $OS(5,6)$ are fixed with a value of zero because the addition of any results in an infeasible solution. This results in two possibilities: $OS(7,8) = 1$ and $OS(8,9) = 0$ (row III) or $OS(7,8) = 0$ and $OS(8,9) = 1$ (row IV). The method then examines the other branch, namely $OS(1,1) = 0$. It then branches on free variable $OS(2,2)$. With $OS(2,2) = 1$, $OS(5,5)$ and $OS(5,6)$ have to be fixed with a value of zero because the addition of either results in an infeasible solution. One more time, two options exist: $OS(7,8) = 1$ and $OS(8,9) = 0$ (row V) or $OS(7,8) = 0$ and $OS(8,9) = 1$ (row VI). The method then explores the branch $OS(1,1) = 0$ and $OS(2,2) = 0$. In row VII, the method bifurcates on free variable $OS(5,5)$. If $OS(5,5) = 1$, all other free variables have to be made equal to zero to avoid infeasibility (row VII). The method then considers the other branch: $OS(1,1) = 0$, $OS(2,2) = 0$ and $OS(5,5) = 0$. In fact, in rows VIII and IX is branching on free variable $OS(5,6)$.

It should be noted that this process is repeated in rows X through XVI for $OS(4,4) = 0$. Furthermore, in row XVII, after OS(6,7) is made equal to zero, no other non-basic variables can be added without violating some constraint. Finally, in row XVIII, even when OS(9,10) is made equal to zero, the solution is still infeasible.

In this example, the local search method went from an infeasible solution (row I) with an objective function value of \$32,602, to a feasible solution with an improved objective function value of \$35,486 (row IX). At this point, the numerical method in figure 8 comes to step 8. It then compares the solutions obtained in sections 4.6.1.1, 4.6.1.2 and 4.6.1.4; and it selects the best available heuristic solution: $\{OS(1,1), OS(2,2), OS(5,6), OS(7,8), OS(9,10)\}$ with an objective function value of \$37,323.

4.6.2 Numerical Example for Minimize-Marginal-Energy-Consumption Philosophy

4.6.2.1 Controlled Marginal Energy Consumption Method

The algorithm in figure 14 first calculates $\Delta P_{j(a)}/I_a$ for the two final-process strategies $OS(7,8)$ and $OS(9,10)$. From the data in table 15 for these operational strategies, these ratios are equal to

0.0202 and 0.0206, respectively, which means that *OS*(9,10) is selected first. After this, the algorithm negatively answers the questions in step 6 and returns to step 2 to select the only remaining final-process strategy, *OS*(7,8), given that its inclusion does not result in the violation of any constraints.

In steps 7 and 8, the following information is generated

Process	PBC
1	0.0082
2	0.0098
3	0.0132
4	0.0199
5	0.0236
6	0.0068
8	0.0045

Since process 5 has the highest PBC value, the two strategies that apply to it – *OS*(5,5) and *OS*(5,6) – become "aspirants" and their ratios are calculated using a ΔT_5 equal to 0.9525,

$$\Delta T_5 P_{5(5)} / I_5 = 114.366 / 4{,}950 = 0.0231$$

$$\Delta T_5 P_{5(6)} / I_6 = 85.762 / 3{,}550 = 0.0241$$

Since *OS*(5,6) exhibits the higher ratio and its addition does not affect feasibility, it is selected. After answering negatively the question in step 15, the algorithm attempts to include *OS*(5,5) but it is eliminated because of an upper-bound constraint violation. The algorithm then reaches step 16 and returns to step 7, which combined with step 8 generates the following information

Process	PBC
1	0.0082
2	0.0098
3	0.0133
4	0.0200
6	0.0068
8	0.0045

Since process 4 now has the highest PBC value, the only strategy that applies to it, *OS*(4,4), is the new "aspirant" with a ratio of 0.0241 based on a ΔT_4 equal to 0.925. Since its inclusion does not exceed any constraints, it is added to the solution.

The algorithm then goes back to step 2 and attempts to select other strategies, but this is not possible due to budget constraint violations, which results in the solution set {$OS(4,4)$, $OS(5,6)$, $OS(7,8)$ and $OS(9,10)$} with an additional total throughput of 326.4 tons/year and an additional total energy consumption of 9,422 kW·h/year.

4.6.2.2 Pareto Optimal Solution Method

The trade-off curve for the hypothetical production process in figure 19 is presented in figure 21. As explained in section 4.4.2, the point (V_{11}, V_{21}) is calculated by setting V_{11} equal to zero and selecting from final-process strategies, $OS(7,8)$ and $OS(9,10)$ in this example, which results in a ΔT of 125 tons/year.

Point (V_{14}, V_{24}) is computed by selecting strategies using the algorithm in figure 16, whose first step yields the values

Operational Strategy	Ratio
$OS(1,1)$	0.0495
$OS(2,2)$	0.0350
$OS(3,3)$	0.0247
$OS(4,4)$	0.0234
$OS(5,5)$	0.0228
$OS(5,6)$	0.0239
$OS(6,7)$	0.0577
$OS(7,8)$	0.0202
$OS(8,9)$	0.0194
$OS(9,10)$	0.0206

The algorithm selects $OS(6,7)$ first since it exhibits the highest ratio and repeats the process, which yields the solution set {$OS(1,1)$, $OS(2,2)$, $OS(5,6)$, $OS(6,7)$, $OS(9,10)$} with a ΔT and a ΔE of 386.0 tons/year and 30,566 kW·h/year.

The solution method then computes

$$V_{22} = 125.0 + \tfrac{1}{3}(386.0 - 125.0) = 211.9$$

$$V_{23} = 125.0 + \tfrac{2}{3}(386.0 - 125.0) = 299.1$$

and uses the algorithm in figure 14 to calculate V_{12} and V_{13} as equal to 3,628 and 8,054 tons/year, respectively. It should be noted that to work out the value of V_{13}, for example, the method

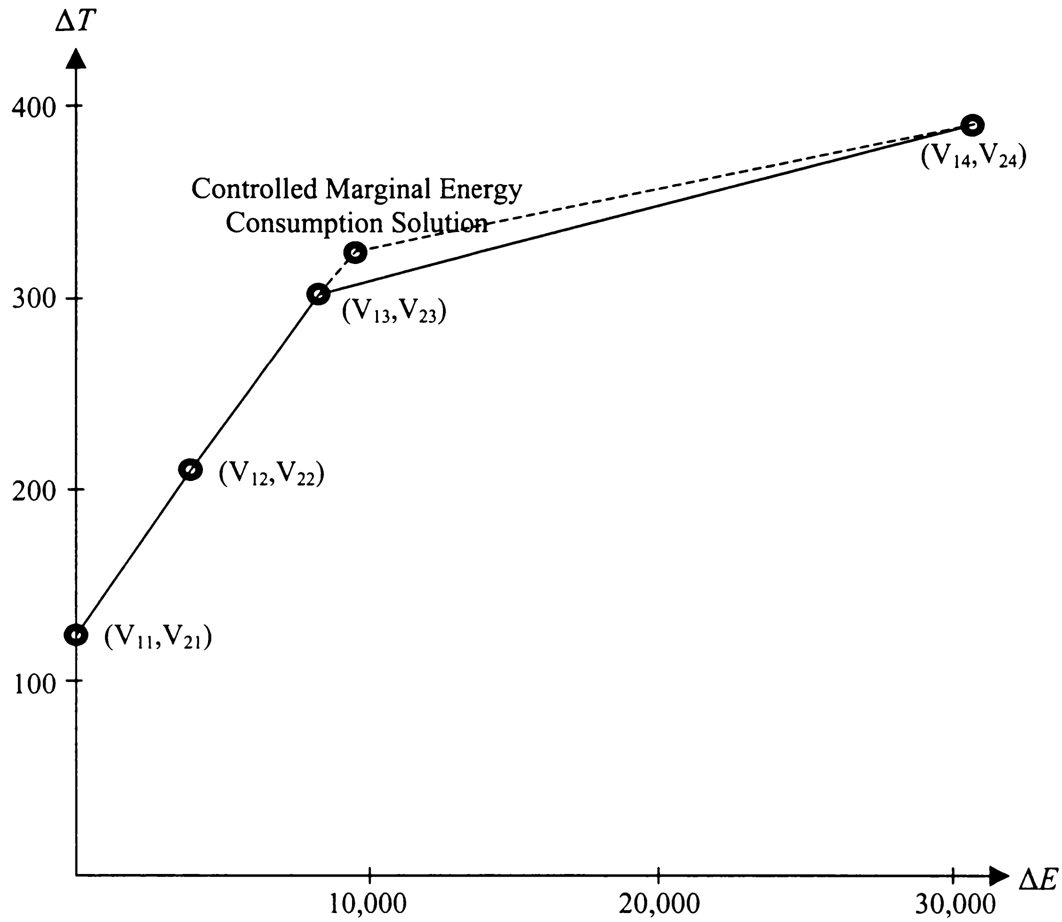


Figure 21. Trade-off Curve for Comprehensive Numerical Example

has to calculate fractional benefits for $OS(4,4)$ after selecting $OS(9,10)$, $OS(7,8)$ and $OS(5,6)$ so that ΔT does not exceed the value of V_{23}, which results in a diminished $\Delta P_{4(4)}$ of 95.5 tons/year.

Finally, figure 21 includes the solution obtained in the previous section (controlled marginal energy consumption method), which is also part of the trade-off curve as explained in section 4.4.2.

CONCLUSION

The purpose of this research was to develop a structured evaluation and optimization methodology for a prototype VCA model created by the ORNL to identify and select new operational strategies/technologies for steel manufacturing plants in order to enhance their performance.

Three major objectives were accomplished during this research. First, an EM model that describes the steel manufacturing process in terms of performance indicators, that adequately explains the marginal changes in outputs that occur per unit changes in inputs at the process step level, and that further illustrates how each process chains together in the production sequence was developed for both manufacturing processes with and without material feedback loops.

Second, EMP models for a number of optimization approaches to search for optimal or pareto-optimal values of the process performance indicators given a set of parameters were developed. These models use the benefit coefficient concept, which is based on the interpretation of the sensitivity parameters, resulting in a procedure that forces us to understand and evaluate the production process' needs and potential first, that allow us to evaluate a diversified group of operational strategies on a leveled playing field based on a specific production process, and that takes into account the global system to assure improved performance for the entire production process.

Third, methods to numerically solve, through a mix of heuristic and optimization techniques, the mathematical programming problems were developed to optimize the manufacturing process' performance in order to achieve the maximum leveraged benefits for the entire enterprise.

A detailed presentation of the theoretical model development process was provided, including in some cases numerical examples to illustrate the mathematical formulations and two comprehensive numerical examples to illustrate and validate the proposed solution methodologies for the enterprise mathematical programming models.

Furthermore, to ensure both the applicability of this research and the attainment of the contributions mentioned in the introduction, a computer program will be written in Matlab to incorporate all the models and solution methods under a single software application, which will be included in the VCA software program to be presented to steel manufacturers willing to participate in a pilot training program leading to the inclusion of the VCA methodology under the U.S. Department of Energy's "Best Practices" program.

Finally, many opportunities for future research derive from this research in terms of both enhancements to the existing models and new models and solution methodologies. For example, the EMP models can be improved by formulating them in such a way that the fractional acceptance of an operational strategy is feasible given the fact that such a case is not allowed in the models presented. As explained in section 3.2.2, if this restriction is relaxed, the estimation of the operational strategy's recurring cash flows is affected although the expected capital investment remains unchanged.

Also, because of the nature of the VCA model, two or more objective functions may need to be considered concurrently and there is a number of additional modeling techniques and solution strategies that can be applied under this scenario (Winston and Albright, 2001). For instance, one of the rarely recognized features of a mathematical programming model is the interplay between objectives and constraints. Once a model has been built, it is extremely easy to convert an objective to a constraint or vice versa. Consequently, the first method for coping with multiple objectives treats all but one objective as constraints and solves the model a number of times while making such changes. The comparison of the different results may suggest a satisfactory solution to the problem or indicate further investigations.

Another popular method known as *goal programming* prioritizes the objective functions, and then tries to satisfy each objective function successively beginning with the highest priority until one of the hard constraints has to be violated. A third common approach is to optimize a suitable linear combination of all the objective functions. It is clearly necessary, under this approach, to attach relative priority weightings to the different objectives.

The fourth technique is Thomas Saaty's *Analytic Hierarchy Process* (AHP). AHP first estimates a weight (priority) for each objective based on pairwise comparisons of objectives, and then estimates a "score" (how well each possibility satisfies each objective) for each possibility based on pairwise comparisons of possibilities for each objective. Finally, the technique calculates for each possibility an overall score that is a weighted sum of the "scores" for that possibility, and the highest overall score indicates the possibility that best satisfies the multiple objectives. Other approaches include interactive multiple-criteria optimization and utility functions.

Another enhancement to the EMP models is possible through the redefinition of the budget constraint. As explained at the beginning of section 3.2.2, the variable *OS(j,a)* represents a 1-1 function between the strategy and the process. However, if a given strategy may be implemented in more than one process, the budget equation

$$\sum_{a=1}^{q}\left(I_a \, OS(j,a)\right) \leq B$$

forces us to incur an I_a capital investment every time the strategy is selected, when it may be possible to get some discounted price from the vendor for additional installations or there may not even be any additional expenses. Obviously, this enhancement would require the definition of a logical relationship to be used by the EMP model.

Furthermore, the heuristic introduced in section 4.3.1 can be improved by redefining the criterion index. For example, an index that measures not only how much return could be expected out of the investment (relative capital return), but also how much available capital the proposed strategy would utilize (capital utilization). The relative capital return component would favor strategies with higher AW_a/I_a ratios, while the capital utilization component would prefer strategies with lower I_a/B ratios. Simply stated, an ideal new operational strategy would necessitate a minimum portion of the available capital and would provide the highest relative surplus. This new index, whose performance would have to be tested, can be expressed as

$$\frac{AW_a/I_a}{I_a/B} = \frac{AW_a \, B}{I_a^{\,2}}$$

Moreover, it may be important for the decision maker to know the net present value, internal rate of return and/or payback period for the optimal or pareto-optimal solution obtained. Better yet, the decision-maker may want these economic performance measures on an after-tax-cash-flow basis, which implies a depreciation schedule for the selected strategies.

Likewise, the decision-maker may want to include additional performance indicators in the EM model, or to consider additional optimization philosophies. The former can be exemplified by some environmental metric; and the latter by objective functions such as maximize/minimize total product throughput and minimize total operating cost, or by final product throughput constraints such as

$$T^{m}{}_{i} \leq Ti_{(j)} + r_{j,0,i}\, \hat{Y}_j \left(\sum_{k=1}^{m} P_{k,j} + \widetilde{P}_j \right) \leq T^{M}{}_{i} \qquad i=1,2,\ldots,n$$

where $T^{m}{}_{i}$ and $T^{M}{}_{i}$ represent minimum and maximum throughput desired for the i^{th} final product, respectively.

Certainly, the foundation for the fourth optimization approach in section 3.1, multiplant optimization, was not provided in this research; and it may require a significant redefinition of the EM and EMP models since material transfers between manufacturing facilities may be present. One possibility is to use multiplant benefit coefficients whose values are approximated using some sort of power measure.

Similarly, both the local search method to improve a solution and the local search method to find a feasible solution can be made more effective through the fine-tuning of some or all of their parameters such as the construction of neighborhoods.

Additionally, the results presented to the decision-maker after the application of the solution methodologies on the EMP models can be superior if *sensitivity analysis* is performed given the fact that many of the input values are estimates in the first place. Sensitivity analysis, also known as post-optimality analysis, investigates the effect on the objective function from changes on the value of a model parameter. Typically, sensitivity analysis considers changes on the objective function coefficients and the right-hand side constraint constants once an optimal solution is found since experience has shown that the information provided by changes on the values of the interior coefficients is far less useful (Murty, 1985). In fact, a method called *Ranging* finds limits (ranges) within which any given coefficient can be changed to have a predicted effect on the solution. It is important to restate that the interpretation of the effect on the objective function is only valid if one coefficient is changed at a time within the permitted ranges.

It is also important to mention that, although the objective of a mathematical programming model is to obtain an optimal solution, practical situations make *stable* solutions -those that will make us very reluctant to change the operating plan if only small changes occur in the parameters of the model- more valuable (Williams, 1985). Obviously, sensitivity analysis is a good way to incorporate this reluctance in the models because of how sensitive an optimal or pareto-optimal solution may be to changes or inaccuracies in the input data.

Although all the constraints in this research have been considered to be *hard* constraints, future EMP models give the decision-maker the option of defining *soft* constraints. Hard constraints are those that cannot be violated, and soft constraints are whose requirement or limit can be altered (Williams, 1985). For example, the constraint below represents a raw material availability limitation,

$$\sum_j a_j x_j \leq b \tag{66}$$

Obviously, if it is worthwhile to buy extra raw material at a high price, Eq. (66) is an unrealistic representation of the situation. In this case, Eq. (66) is rewritten as

$$\sum_j a_j x_j - u \leq b$$

and u is given a suitable positive (negative) coefficient c for a minimization (maximization) problem, b would represent a raw material availability that could be expanded to $b+u$ at a cost cu if the optimization procedure found this to be desirable. It may also be advisable to assign a simple upper bound to the surplus variable u to prevent an increase beyond a specified amount.

If Eq. (66) were a '≥' constraint, the hard constraint could be made soft with a slack variable such as

$$\sum_j a_j x_j + u \geq b$$

and if Eq. (66) were a equality constraint, it would be possible to allow b to be overreached or under reached by modeling it as

$$\sum_j a_j x_j + u - v = b$$

and giving u and v appropriate coefficients in the objective function. It should be noted that either u or v must be zero in the optimal solution.

A very considerable final enhancement that can be made to the VCA model is the addition of the final product mix optimization problem since this research kept the original final product mix unchanged. If this problem is added, the variable $r_{j,k,i}$ becomes a decision variable and three approaches are possible:

a) Optimize the final product mix first and then apply the EMP models,
b) Apply the EMP models first and then optimize the mix, or
c) Redefine the EMP models so that the mix is optimized as strategies are selected.

LIST OF REFERENCES

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Aarts, Emile and Jan Karel Lenstra. *Local Search in Combinatorial Optimization*. John Wiley & Sons: New York, NY, 1997.

Agrawal, S. C. "On Integer Solution to Quadratic Programs by the Branch and Bound Technique" (In Spanish). *Estadística de Investigación Operativa* **1975**, Madrid, España.

American Iron and Steel Institute (AISI), Steel Manufacturers Association (SMA), and U.S. Department of Energy. *Steel Industry Technology Roadmap*. Washington, D.C., 1998. Available on-line at www.steel.org/mt/roadmap/roadmap.htm.

American Iron and Steel Institute (AISI) and Steel Manufacturers Association (SMA). *Steel: A National Resource for the Future*. Washington, D.C., 1995.

Bjorndal, M. H. et al. "Some Thoughts on Combinatorial Optimization." *European Journal of Operational Research* **1995**, vol. 83, 253-270.

Cabot, A. V. and S. S. Erenguc. "A Branch and Bound Algorithm for Solving a Class of Nonlinear Integer Programming Problems." *Naval Research Logistics Quarterly* **1986**, vol. 33, 559-567.

Christofides, Nicos et al. *Combinatorial Optimization*. John Wiley & Sons: New York, NY, 1979.

Cooper, Mary. "A Survey of Methods for Pure Nonlinear Integer Programming." *Management Science* **1981**, vol. 27, no. 3, 353-361.

Couretas et al. "Capacity Analysis for Mixed Technology Production." *International Journal of Production Research* **2001**. vol. 39, no. 2, 163-184.

Cyert, Richard and R. Fruehan. *Meeting the Challenge: U.S. Industry Faces the 21st Century: The Basic Steel Industry*. U.S. Department of Commerce: Washington, D.C., 1996. Available on-line at www.ta.doc.gov/reports/steel/cd91a.pdf.

Dadebo, Solomon Aikins. "Dynamic Optimization and Nonlinear Control of Chemical Processes." Ph.D. Dissertation, Queen's University at Kingston, Canada, 1996.

Dantzig, G. B. "Discrete Variable Extremum Problems." *Operations Research* **1957**, vol. 5, 266-277.

Dantzig, George and Mukund Thapa. *Linear Programming 1: Introduction*. Springer: New York, NY. 1997.

Eiselt, H. A. and C.-L. Sandblom. *Integer Programming and Network Models*. Springer: Berlin, Germany, 2000.

Farrager, Edward, Robert Kleiman and Anandi Sahu. "Current Capital Investment Practices." *The Engineering Economist* **1999**, vol. 44, no. 2, 137-150.

Fleischer, G.A. *Introduction to Engineering Economy*. PWS Publishing Company: Boston, MA, 1994.

Fortuin, Leonard, Paul van Beek and Luck Van Wassenhove. *OR at Work: Practical Experiences of Operational Research*. Taylor & Francis: London, England, 1996.

Gisdvold, K. M. and J. Moe. "A Method for Nonlinear Mixed-Integer Programming and Its Applications to Design Problems." *Journal of Engineering for Industry* **1972**, May.

Glover, Fred. "Improved Linear Integer Programming Formulations of Nonlinear Integer Problems." *Management Science* **1975**, vol. 22, no. 4, 455-460.

Glover, Fred. "New Approaches for Heuristic Search. A Bilateral Linkage with Artificial Intelligence." *European Journal of Operational Research* **1986**, vol. 39, 119-130.

Glover, Fred, Eric Taillard and Dominique de Werra. "Tabu Search." *Annals of Operations Research* **1993**, vol. 41, 3-28.

Glover, Fred and Manuel Laguna. *Tabu Search*. Kluwer Academic: Boston, MA, 1997.

Gordon, L. and G. Pinches. *Improving Capital Budgeting Systems: A Decision Support System Approach*. Addison-Wesley: Reading, MA, 1984.

Greenberg, Harold and Robert Hegerich. "A Branch Search Algorithm for the Knapsack Problem." *Management Science* **1970**, vol. 16, 327-332.

Gupta, O. K. "Branch and Bound Experiments in Nonlinear Programming." Ph.D. Dissertation, Purdue University, West Lafayette, Indiana, 1980.

Hansen, P. "The Steepest Ascent, Mildest Descent Heuristic for Combinatorial Programming." Congress on Numerical Methods in Combinatorial Optimization, Capry, Italy, 1986.

Horowitz, Ellis and Sartaj Sahni. "Computing Partitions with Applications to the Knapsack Problem." *Journal of the Association of Computing Machinery* **1974**, vol. 21, 277-292.

Hudson, P. D. "Improving the Branch and Bound Algorithms for the Knapsack Problem." *Queen's University Research Report*. Belfast, Ireland, 1977.

Jawahir, Keith. "Continuous Caster Scheduling: An Optimization Approach Using Column Generation." Ph.D. Dissertation, University of Toronto, Canada, 1998.

Junno, Seija Maria Liisa. "Optimization of Steel Mill Production." Ph.D. Dissertation, Oulun Liopisto University, Finland, 1989.

Kaplan, S. "Solution of the Lorie-Savage and Similar Integer Programming Problems by the Generalized Lagrange Multiplier Method." *Operations Research* **1966**, vol. 14, 1130-1135.

Karabal, Nejat, James Bean and Jack Lohman. "Solving Large Replacement Problems with Budget Constraints." *The Engineering Economist* **2000**, vol. 45, no. 4, 290-308.

Kolesar, P. J. "A Branch and Bound Algorithm for the Knapsack Problem." *Management Science* **1967**, vol. 13, 723-735.

Korte, Bernhard and Jens Vygen. *Combinatorial Optimization: Theory and Algorithms*. Springer: New York, NY, 2000.

Kreher, Donald and Douglas Stinson. *Combinatorial Algorithms: Generation, Enumeration, and Search*. CRC Press: Boca Raton, FL, 1999.

Lang, A. H. and A. G. Doig. "An Automatic Method of Solving Discrete Linear Programming Problems." *Econometrica* **1960**, vol. 28, 496-520.

Lau, H. T. *Combinatorial Heuristic Algorithms with Fortran*. Springer-Verlag: Berlin, Germany, 1986.

Lee, Won Jun. "Branch and Bound Methods for Solving a Class of Nonlinear Integer Programming Problems." Ph.D. Dissertation, Indiana University, Bloomington, Indiana, 1989.

Li, Hong and Shang. "Integrated Model for Production Planning in a Large Iron and Steel Manufacturing Environment." *International Journal of Production Research* **2001**, vol. 39, no. 9.

Lorie, J. and L. J. Savage. "Three Problems in Capital Rationing." *Journal of Business* **1955**, vol. 28, 229-239.

Lopez, Leo, Michael Carter and Michael Gendreau. "The Hot Strip Mill Production Scheduling Problem: A Tabu Search Approach." *European Journal of Operational Research* **1998**, vol. 106, 317-335.

Martello, Silvano and Paolo Toth. "An Upper Bound for the Zero-One Knapsack Problem and a Branch and Bound Algorithm." *European Journal of Operational Research* **1977**, vol. 1, 169-175.

Martello, Silvano and Paolo Toth. *Knapsack Problems: Algorithms and Computer Implementations*. John Wiley & Sons: New York, NY, 1990.

Moder, Joseph and Salah Elmaghraby. *Handbook of Operations Research: Foundations and Fundamentals. Volume 1*. Van Nostrand Reinhold: New York, NY, 1978.

Murty, Katta G. *Linear and Combinatorial Programming*. Robert E. Krieger Publishing: Malabar, FL, 1985.

Park, Chan and Gunter Sharp-Bette. *Advanced Engineering Economics*. John Wiley & Sons: New York, NY, 1990.

Pfaffenberger, Roger and David Walker. *Mathematical Programming for Economics and Business*. Iowa State University Press: Ames, Iowa, 1976.

Pinches, G. Myopia. "Capital Budgeting and Decision Making." *Financial Management* Autumn **1982**, 6-19.

Pirlot, Marc. "General Local Search Methods." *European Journal of Operational Research* **1996**, vol. 92, 493-511.

Rardin, Ronald. *Optimization in Operations Research*. Prentice Hall: Upper Saddle River, NJ, 1998.

Reeves, Colin. *Modern Heuristic Techniques for Combinatorial Problems*. John Wiley & Sons: New York, NY, 1993.

Salkin, Harvey and Kamlesh Mathur. *Foundations of Integer Programming*. North-Holland: New York, NY, 1989.

Shao, Yuehjen Eric. "A Real-Time Stochastic Process Control System for Process Manufacturing." Ph.D. Dissertation, Rensselaer Polytechnic Institute, 1993.

Solow, Daniel. *Linear Programming: An Introduction to Finite Improvement Algorithms*. North-Holland: New York, NY, 1984.

Sonntag, Richard, Claus Borgnakke and Gordon Van Wylen. *Fundamentals of Thermodynamics*. Wiley: New York, NY, 1998. 5th ed.

Sullivan, James, James Bontadelli and Elin Wicks. *Engineering Economy*. Prentice Hall: Upper Saddle River, NJ, 2000. 11th ed.

Tobin, Roger. "A Fast Interactive Solution Method for Large Capital Expenditure Selection Problems." *European Journal of Operational Research* **1999**, vol. 116, 1-15.

Wei, Chiu-Chi, Chie-Bein Chen and Chih-Hung Tsai. "An Efficient Approach to Prioritize Projects Under Budget Constraints." *The Engineering Economist* **1999**, vol. 44, no 3, 261-275.

Weingartner, H. M. *Mathematical Programming and the Analysis of Capital Budgeting Problems*. Prentice-Hall: Englewood Cliff, NJ, 1963.

West, Timothy. "A More Efficient Branch and Bound Algorithm for Nonlinear Integer and Discrete Programming." M.S. Thesis, University of Missouri, Columbia, Missouri, 1988.

Williams, Hillary P. *Model Building in Mathematical Programming*. John Wiley & Sons: Chichester, NY, 1985. 2nd ed.

Winston, Wayne L. *Operations Research: Applications and Algorithms*. Duxbury Press: Boston, MA, 1987.

Winston, Wayne and S. Christian Albright. *Practical Management Science*. Duxbury: Pacific Grove, CA, 2001. 2nd ed.

Wolsey, Laurence. *Integer Programming*. John Wiley & Sons: New York, NY, 1998.

Yu, Gang. *Industrial Applications of Combinatorial Optimization*. Kluwer Academic Publishers: Boston, MA, 1998.

APPENDIX

APPENDIX

PROTOTYPE VCA MODEL

The VCA model first identifies potential benefits for all processes than can be derived from new technologies or operational strategies. Under the assumption that a given operational strategy will rather affect the operational efficiencies around the current operating point than change the way steel is made, the Taylor series approximation is used to estimate potential process gains as follows.

If $F(\mathbf{X}) \in \mathbb{R}^m$ is the function of a set of performance indicators, $\mathbf{X} \in \mathbb{R}^n$ is a vector of independent operating variables, and $\mathbf{X}_C$ and $\mathbf{X}_N$ represent the current and new operating points, respectively; then the first two terms of the Taylor series approximation can be used to estimate

$$F(\mathbf{X}_N) = F(\mathbf{X}_C) + \mathbf{J}(\mathbf{X}_C)(\mathbf{X}_N - \mathbf{X}_C)$$

where $\mathbf{J}(\mathbf{X}_C)$ denotes the mxn Jacobian matrix of first partial derivatives of $F(\mathbf{X})$ at $\mathbf{X}_C$ and

$$[\mathbf{J}(\mathbf{X}_C)]_{ij} = \frac{\partial F_i(\mathbf{X}_C)}{\partial X_j}, \quad i = 1,\ldots, m \text{ and } j = 1,\ldots, n$$

Since $F(\mathbf{X})$ is unknown, the VCA model then estimates the value of the differential terms as slope ratios around the current operating point since these ratios -henceforth known as sensitivity parameters- can be more easily calculated from available process and economic data. In other words,

$$\frac{\partial F_i(\mathbf{X})}{\partial X_j} \approx \frac{\Delta F_i(\mathbf{X})}{\Delta X_j}$$

The sensitivity parameters are invariant economic and operational indicators that quantify the impact of any proposed technology in terms of material throughput, energy usage, efficiency change, and costs. Next, the model will develop a set of coupled equations from these parameters that links the entire production system together so that total enterprise impacts can be calculated. Figure A-1 illustrates the 6 steps included in the VCA model.

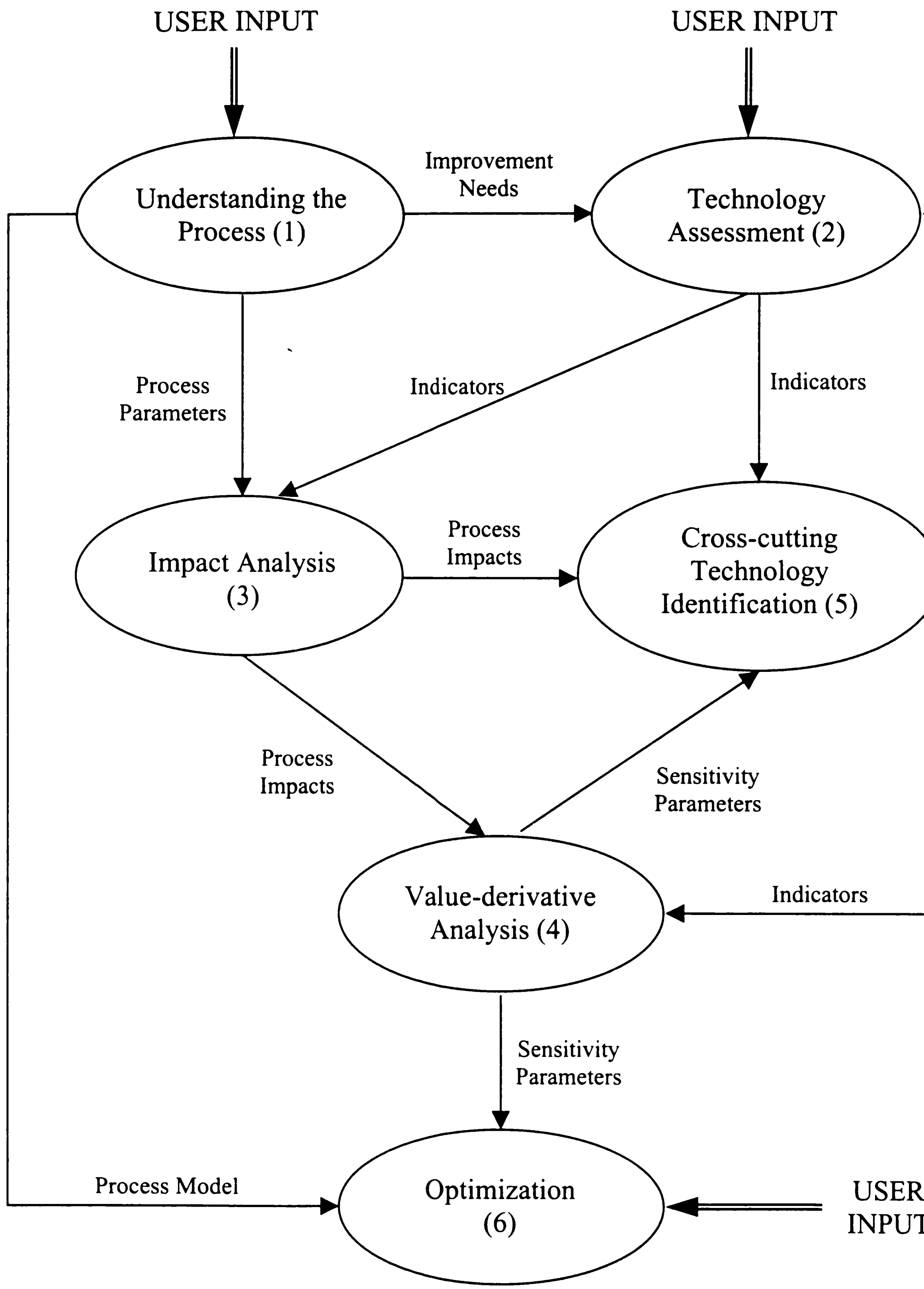


Figure A-1. Process Flow Diagram for the VCA Model

The first step is *Understanding the Process*. A requirement for conducting a VCA is to develop a complete understanding of the process: its product flow, procedural steps, energy use, maintenance and operational procedures, measurements of performance, etc. This first step generates three outcomes: a process model in the form of a process flow diagram, a set of process parameters, and a group of improvement needs.

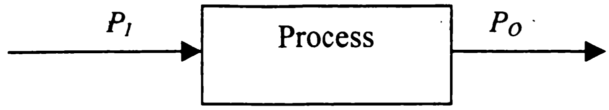
The second step, *Technology Assessment*, basically defines the physical attributes of the process that need to be controlled, measured, and quantified; and relates them to technologies, either existing (enabling) or being developed (emerging), that the users consider important to improving their plant performance. Each proposed technology is listed along with its current operating profile, desired profile and figures of merit, which are used to calculate the indicators. For an explanation of the calculations embedded in the VCA model, please see the section titled "Calculating Indicators, Process Impacts and Sensitivity Parameters" at the end of this appendix.

With the process parameters and indicators as inputs, the *Impact Analysis* step is then used to estimate process impacts. These process impacts, along with the indicators, become the information sources to calculate the value-differential sensitivity parameters during the *Value-derivative Analysis* step. The fifth step, *Cross-cutting Technology Identification*, is a tool to identify technologies by function and level of maturation that can be applied across the steel industry. Finally, the *Optimization* step uses the process model and sensitivity parameters to create a set of coupled equations to search for an optimal or pareto-optimal solution for a given set of constraints.

It should be noted that user input is critical during the Understanding the Process, Technology Assessment and Optimization steps. Moreover, a user will normally represent a group of managers and technical representatives from each of the individual steel-making processes.

Calculating Indicators, Process Impacts and Sensitivity Parameters

The steel-making process can be represented as a sequence of connected processes with specific process parameters, indicators, process impacts and sensitivity parameters that are calculated as indicated below.



Process Parameters

P_I = Product input (tons/year)

P_O = Product output or throughput (tons/year)

P_O (This process) = P_I (Subsequent process)

P_I (This process) = P_O (Preceding process)

Y = Process yield (no units)

$$Y = \frac{P_O}{P_I} \times 100$$

Utilization = Level of production capacity currently used (no units)

$$\text{Utilization} = \frac{\text{current } P_I}{\max P_I}$$

Indicators

ΔP = Additional product resulting from operational improvements within the process (tons/year)

$$\Delta P = \Delta P_O$$

PDC_t = Production cost per ton (\$/ton)

ΔPDC = Production cost savings (\$/year)

$$\Delta PDC = PDC_t \times \Delta P$$

PDC_t is the cost that a ton of material that arrives to this process accumulated as it moved through the preceding processes; and, consequently, ΔPDC represents the potential cost savings due to ΔP.

Process Impacts

E_t = Energy per ton (kW·h/ton)

ΔE = Energy savings due to ΔP (kW·h/year)

$$\Delta E = \Delta P \times E_t$$

η = Efficiency (no units)

$$\Delta \eta = 1 - \frac{\text{Tons}_{\text{lost}} - \text{Tons}_{\text{gained}}}{\text{Tons}_{\text{lost}}} = 1 - \frac{P_I - P_O - \Delta P}{P_I - P_O} = \frac{\Delta P}{P_I - P_O}$$

$\Delta \eta$ represents the percentage of material lost, which is the difference between P_I and P_O, that is recovered due to operational improvements.

F_t = Fixed Cost per Ton ($/ton)

V_t = Variable Cost per Ton ($/ton)

ΔPCC = Process Cost Savings ($/year)

$$\Delta PCC = (F_t + V_t)\,\Delta P$$

PCC is the cost that a ton of material that arrives to this process will incur as it goes through the process regardless of process yield; and consequently, ΔPCC represents the potential cost savings due to ΔP.

Sensitivity Parameters

$$\frac{\Delta E}{\Delta \eta} = \frac{\Delta P \times E_t}{\frac{\Delta P}{P_I - P_O}} = (P_I - P_O)\,E_t$$

$$\frac{\Delta \eta}{\Delta C} = \frac{\frac{\Delta P}{P_I - P_O}}{\Delta PDC + \Delta PCC} = \frac{\Delta P}{(PDCt \times \Delta P + (F_t + V_t)\,\Delta P)\,(P_I - P_O)} = \frac{1}{(PDC_t + F_t + V_t)\,(P_I - P_O)}$$

$$\frac{\Delta C}{\Delta P} = \frac{\Delta PDC + \Delta PCC}{\Delta P} = PDC_t + F_t + V_t$$

$$\frac{\Delta P}{\Delta E} = \frac{1}{E_t}$$

VITA

Jaime Torres was born in Bucaramanga, Colombia, South America in 1969. He went to elementary, junior high, and high school at Colegio Calasanz in Bogotá, Colombia. He received his Bachelor of Science in Industrial Engineering from Pontificia Universidad Javeriana in Bogotá, Colombia in October 1991 while working for Cogra Lever (Unilever) in the sales department. He also had the highest GPA of the graduating class, and a chapter of his thesis was published by *Avicultores* – the main publication of the Colombian Poultry Business Federation.

He moved to the United States in 1992, and obtained his Master of Science in Industrial Engineering from University of Tennessee, Knoxville in 1994. He later pursued his doctorate in Engineering Science while working as a graduate teaching associate at the University of Tennessee, Knoxville, and as a manufacturing engineer for Tennessee WaterCraft (Yamaha Motor Corporation USA) and Yale Commercial Locks and Hardware. The doctoral degree was granted by University of Tennessee, Knoxville, in May 2002.

Jaime is currently working for Yale Commercial Locks and Hardware in Lenoir City, Tennessee, as profit center manager for the exit device product line.